Chapter 6
Work, energy and power

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COURSE TOPICS:

6.1 Work
6.2 Kinetic energy
6.3 Potential energy and conservative forces
6.9 Power

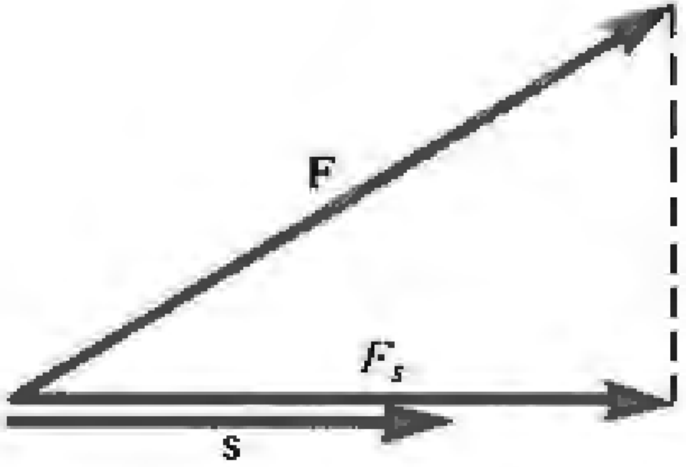
6.1 Work:

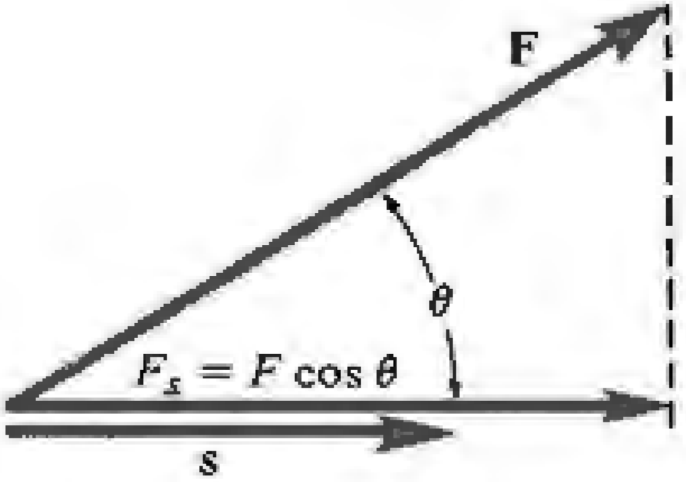
- The concept of work plays a fundamental role in the analysis of many mechanical problems.
- Suppose that an object is displaced a distance $\boldsymbol{s}$, and that a force $\boldsymbol{F}$ acting on the object has a constant component F_s, along $\boldsymbol{s}$. Then the work done by the force is defined as the product of the force component and the displacement,

$$W = F_s s = Fs \cos \theta$$

- The S.I. unit of work is the joule (J).
- Since work has dimensions of force times distance, a joule is a newton meter:

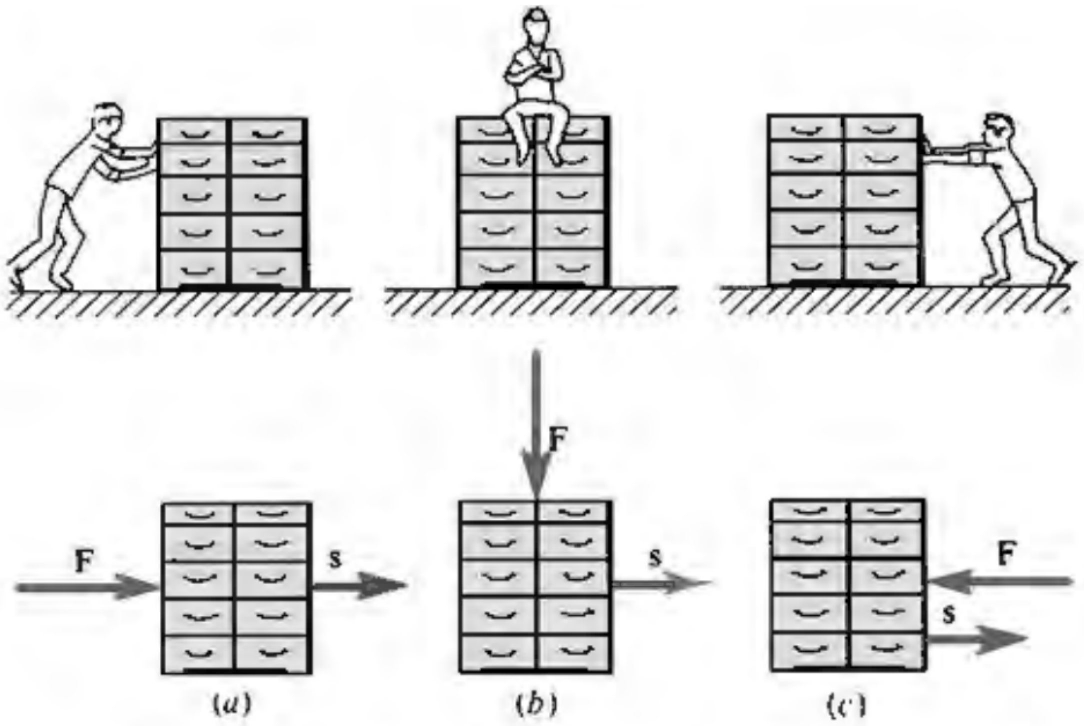
$$1\,J = 1\,N.m = 1\,kg.m^2/s^2$$





6.1 Work:

Example 6.1.
A 600-N force is applied by a man to a dresser that moves 2 m. Find the work done if the force and displacement are (a) parallel; (b) at right angles; (c) oppositely directed.



Answer:

(a) When $\boldsymbol{F}$ and $\boldsymbol{s}$ are parallel, $\cos\theta = \cos 0 = 1$, and then

$$W = \mathbf{F}\mathbf{s}\cos\theta = (600\text{ N})(2\text{ m})(1) = 1200\text{ J}$$

(b) When $\boldsymbol{F}$ is perpendicular to $\boldsymbol{s}$, $\cos\theta = \cos 90° = 0$ and then

$$W = Fs\cos\theta = (600\ N)(2\ m)(0) = 0$$

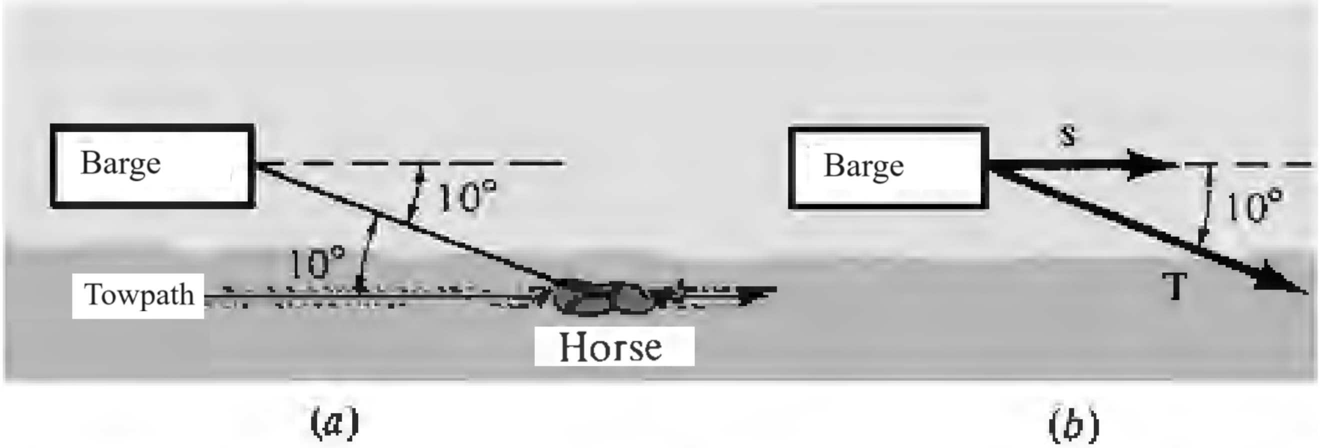
(c) When $\boldsymbol{F}$ and $\boldsymbol{s}$ are opposite, $\cos\theta = \cos 180° = -1$

$$W = Fs\cos\theta = (600\text{ N})(2\text{ m})(-1) = -1200\text{ J}$$

6.1 Work:

Example 6.2.

A horse pulls a barge along a canal with a rope in which the tension is 1000 N. The rope is at an angle of 10° with the towpath and the direction of the barge. (a) How much work is done by the horse in pulling the barge 100 m upstream at a constant velocity? (b) What is the net force on the barge?



Answer

(a) The work done by the constant force $\boldsymbol{T}$ in moving the barge a distance is given by

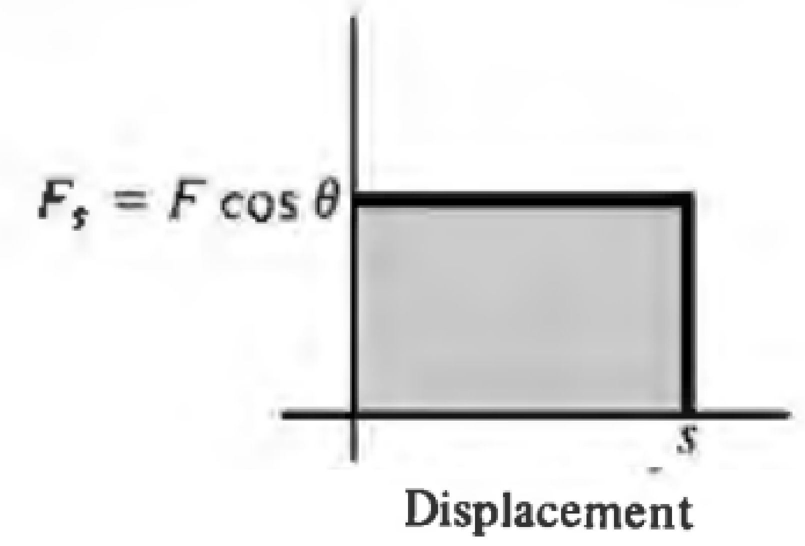
$$W = T\, s\, \cos\theta = (1000\ \mathrm{N})(100\ \mathrm{m})(\cos 10°) = 9.85 \times 10^4\ \mathrm{J}$$

(b) Since the barge moves at a constant velocity, the sum of all the forces on it must be zero. There must be another force acting that is not shown in the figure, a force exerted on the barge by the water that is equal in magnitude and opposite to $\boldsymbol{T}$.
In this example the net work done by all the forces acting on the barge is zero because the net force is zero. The work done by the force due to the water is -9.85×10^4 J . In other words, the barge does 9.85 $\times\ 10^4$ J of work on the water.

6.1 Work:

- Note that if we graph $\boldsymbol{F}$, for a constant force versus s for a given motion, then the rectangular area under the curve is the product $F_s s$. This equals the work done by the force.

- This result is true in general: **the area under the graph of $\boldsymbol{F}$, versus $\boldsymbol{s}$ for any motion is the work done**



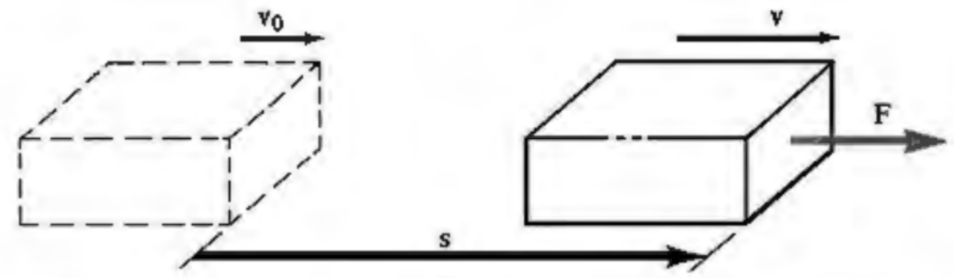
6.2 Kinetic energy:

- The kinetic energy of an object is a measure of the work an object can do by virtue of its motion.
- As we show in the following, the translational kinetic energy of an object of mass m and velocity v is

$$K = \frac{1}{2}mv^2$$

- Consider an object of mass m subjected to a constant force $\boldsymbol{F}$. The object accelerates from $\boldsymbol{v}_0$ to $\boldsymbol{v}$ after moving a distance $\boldsymbol{s}$ parallel to $\boldsymbol{F}$. Since its acceleration $\boldsymbol{a}$ is constant then

$$v^2 = v_0^2 + 2as$$



- Multiplying by $m/2$, this becomes

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0^2 + mas$$

- Using Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, the work done by the force $\boldsymbol{F}$ is $W = Fs = mas$. With $K = \frac{1}{2}mv^2$ and $K_0 = \frac{1}{2}mv_0^2$ are the initial and final kinetic energies, then:

$$\boldsymbol{K = K_0 + W} \quad \textbf{or} \quad \mathbf{W} = \boldsymbol{\Delta K = K - K_0}$$

Therefore, the final kinetic energy is equal to the initial kinetic energy of the object plus the work done on it. Or the net work done on the object is equal to the change in its kinetic energy

6.2 Kinetic energy:

Example

A ball of mass m is moving with velocity v. (a) What is the kinetic energy of the ball? (b) If the speed of the ball is doubled, what is the new kinetic energy and what is the work done of the ball by the applied force

Answer

(a) $v_0 = v$, so $K_0 = \frac{1}{2}mv^2$

(b) The velocity now is $2v$, so

$$K = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4K_0$$

$$\mathrm{W} = \Delta K = K - K_0 = 4K_0 - K_0 = 3K_0 = \frac{3}{2}mv^2$$

6.2 Kinetic energy:

Example 6.3

A woman pushes a toy car, initially at rest, toward a child by exerting a constant horizontal force $\boldsymbol{F}$ of magnitude 5 N through a distance of 1 m. (a) How much work is done on the car? (b) What is its final kinetic energy? (c) If the car has a mass of 0.1 kg, what is its final speed? (Assume no work is done by frictional forces.)

Answer

(a) The force the woman exerts on the car is parallel to the displacement, so the work she does on the car is

$$W = Fs = (5 \text{ N})(1 \text{ m}) = 5 \text{ J}$$

The initial kinetic energy Ks is zero, so the final kinetic energy of the car is

$$K = K_0 + W = 0 + 5 \text{ J} = 5 \text{ J}$$

(b) The final kinetic energy is $K = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5 \text{ J})}{0.1 \text{ kg}}} = 10 \text{ m s}^{-1}$$

6.2 Kinetic energy:

Example 6.4

In the preceding example the woman releases the toy car with a kinetic energy of 5 J. It moves across the floor and reaches the child who stops the car by exerting a constant force $\boldsymbol{F}'$ opposite to its motion. The car stops in 0.25 m. Find $\boldsymbol{F}'$ if no work is done on the car by frictional forces

Answer

While the car is moving toward the child, no work is done on the car, and its kinetic energy remains 5 J until it reaches the child. The initial kinetic energy $K_0 = 5\ J$, and the final kinetic energy $\mathrm{K} = 0$, since the car comes to rest, so

$$W = K - K_0 = 0 - 5\text{ J} = -5\text{ J}$$

Since $\boldsymbol{F}'$ is opposite to $\boldsymbol{s}'$, the work done is

$$W = -F's\ .$$

Thus

$$F' = -\frac{W}{s'} = -\frac{-5\text{ J}}{0.25\text{ m}} = 20\text{ N}$$

6.3 Potential energy:

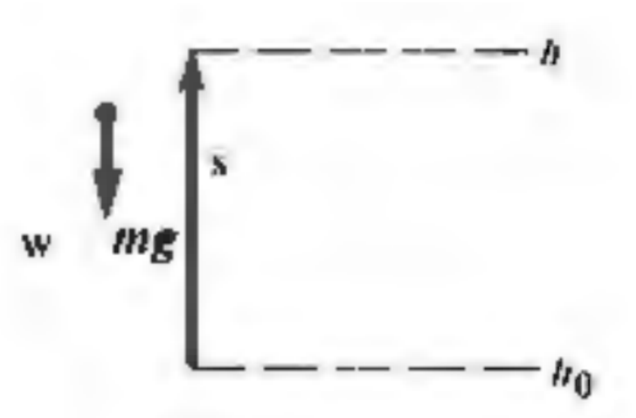
- The work-energy relation ($W = \Delta K = K - K_0$) includes the work done by all the forces acting on the object.
- Introduce another form of energy called **potential energy**.
    - Forces that can be dealt with in this way are called conservative forces.

- Consider a ball thrown straight up. Its velocity decreases steadily as it rises. From the point of view of this chapter, the gravitational force mg is doing negative work, since it is opposite to the displacement, and the kinetic energy is diminishing correspondingly. Once the ball starts to come down, the gravitational force does an equally large amount of positive work, and the kinetic energy returns to its initial value once the ball returns to its starting point.

- Alternatively, we can think of the rising ball losing kinetic energy and gaining potential energy. This potential energy is converted back into kinetic energy when the ball falls.

- In general, potential energy is energy associated with the position or configuration of a mechanical system.
    - The potential energy can, at least in principle, be converted into kinetic energy or used to do work.

6.3 Potential energy:



- Consider a ball rises from an initial height hs to a height h.
- The gravitational force $\boldsymbol{mg}$ is opposite in direction to the displacement $s = h - h_0$, so the work done is negative:

$$W(\text{grav}) = -mg(h - h_0)$$

- The potential energy increases in this situation, and the change in potential energy is positive

$$\Delta\mathcal{U} = \mathcal{U} - \mathcal{U}_0 \text{ is } \textit{positive}$$

- The magnitude of $\Delta\mathcal{U}$ is defined to be equal to the magnitude of $W(\text{grav})$, so we write

$$\mathcal{U} - \mathcal{U}_0 = -W(\text{grav})$$

  - The minus sign takes care of the differences in sign

- Using our expression for $W(\text{grav}) = -mg(h - h_0)$, we obtain

$$\mathcal{U} - \mathcal{U}_0 = mg(h - h_0)$$

- We define the potential energies themselves at h and h_0, resptively, by

$$\mathcal{U} = mgh \quad \text{and} \quad \mathcal{U}_0 = mgh_0$$

6.3 Potential energy:

- By the work-energy principle,

$$K = K_0 + W(\text{grav}) = K_0 - (\mathcal{U} - \mathcal{U}_0).$$

- So, we have the important result

$$\boxed{K + \mathcal{U} = K_0 + \mathcal{U}_0 \qquad (W_a = 0)}$$

when there is no work done by applied forces, the total mechanical energy is constant or conserved.

- The notation $W_a = 0$ reminds us that the work done by applied forces is zero; only the gravitational force is doing work here.
- The sum of the kinetic energy and the potential energy is called the total mechanical energy,

$$\boxed{E = K + \mathcal{U}}$$

- If applied forces also do work, then we must generalize the above equation to include this work, W_a. Then we have

$$\boxed{K + \mathcal{U} = K_0 + \mathcal{U}_0 + W_a}$$

or

$$\boxed{E = E_0 + W_a}$$

The final mechanical energy $E = K + \mathcal{U}$ is equal to the initial mechanical energy $E_0 = K_0 + \mathcal{U}_0$ plus the work done by the applied forces W_a

6.3 Potential energy:

Comments

- The work done by the gravitational force is still $-\mathrm{mg}(h - h_0)$, so the potential energy change $\Delta\mathcal{U} = \mathcal{U} - \mathcal{U}_0$ depends only on the difference in heights.

- $\Delta\mathcal{U} = \mathcal{U} - \mathcal{U}_0 = \mathrm{mg}(h - h_0)$ remains the same even measure from any convenient reference level: the ground, the top of a building, and so on

- We must remember that $E = E_0 + W_a$, is just another form of our original work-energy principle, $K = K_0 + W$, which involves the work done by all the forces. In $E = E_0 + W_a$, the work done by the gravitational forces is still present, but it is now separated from the work W_a, done by the other forces and taken into account through the potential energy.

6.3 Potential energy:

Example

A ball is at a height h_0. If it is lifted to double that height, what is the new potential energy and what is the work done by gravity

Answer

$\mathcal{U}_0 = mgh_0$,

$h = 2h_0$, so $\mathcal{U} = mgh = mg(2h_0) = 2mgh_0 = 2\mathcal{U}_0$

$W(\text{grav}) = -\text{mg}(h - h_0) = -mgh_0$

6.3 Potential energy:

Example

A ball of mass 0.2 kg is dropped from rest from a height 10 m. (a) How much work is done by the gravitational force on the ball as it falls down the 10 m? (b) What is the speed of the ball after falling the 10 m? (c) What is the speed of the ball when it was 2 m above the ground?

Answer:

(a) We are given: $h_0 = 10\,m, h = 0\,m, m = 0.2\,kg$

$$W(\text{grav}) = -\text{mg}(h - h_0) = -(0.2\,kg)(9.8\,m/s^2)(0 - 10\,m) = 19.6\,J$$

(b) $K + \mathcal{U} = K_0 + \mathcal{U}_0$

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2 + mgh_0$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgh_0$$

$$v = \sqrt{2gh_0} = \sqrt{(2)(9.8\,m/s^2)(10\,m)} = 14\,m/s$$

(c) $K + \mathcal{U} = K_0 + \mathcal{U}_0$

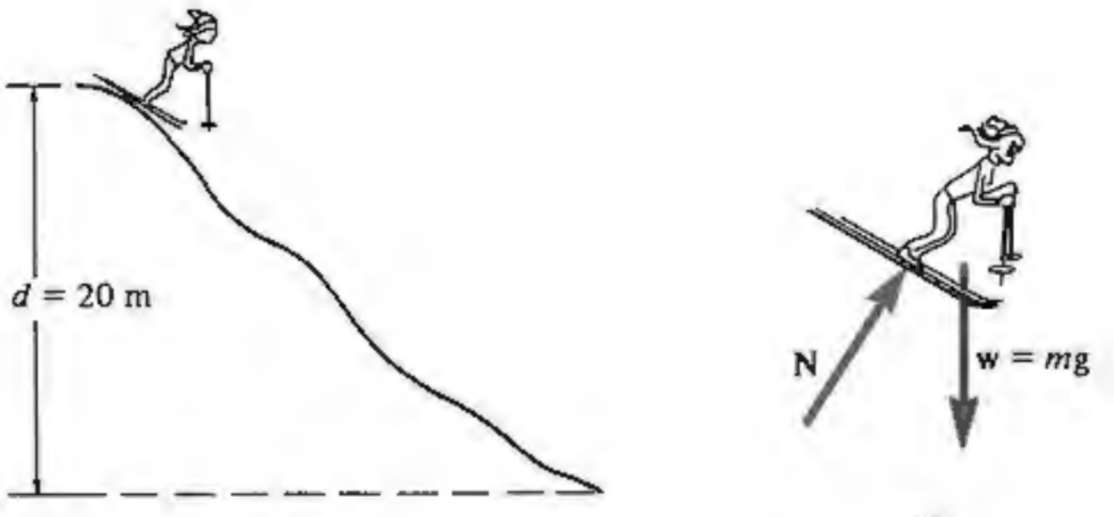
$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2 + mgh_0 \quad \Longrightarrow \quad \frac{1}{2}v^2 + gh = 0 + gh_0$$

$$v = \sqrt{(2)(9.8\,m/s^2)(10\,m - 2\,m)} = 12.5\,m/s$$

6.3 Potential energy:

Example 6.5

skis from rest down a hill 20 m high. If friction is negligible, what is her speed at the bottom of the slope?



(a) (b)

Answer:

Since we can choose the reference level for measuring potential energy as we wish, we choose the bottom of the slope as the level at which $\mathcal{U} = 0$.

The kinetic energy at the top is $K_0 = 0$, since she starts from rest and her final potential energy is $\mathcal{U} = 0$

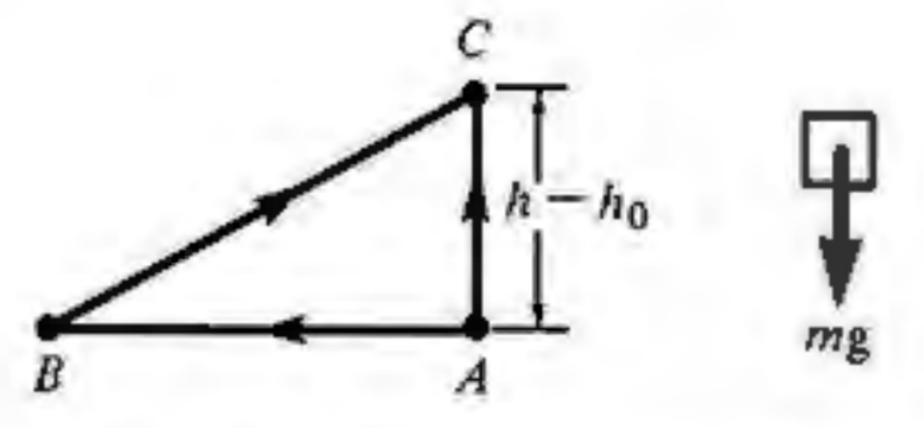
$$K + \mathcal{U} = K_0 + \mathcal{U}_0$$

$$\tfrac{1}{2}mv^2 + 0 = 0 + mgd$$

$$v = \sqrt{2gd} = \sqrt{2(9.8\ \text{m s}^{-2})(20\ \text{m})}$$
$$= 19.8\ \text{m s}^{-1}$$

6.3 Potential energy:
Conservative forces:

- The gravitational force has the interesting property that when an object moves from one point to another, the work done by the force does not depend on the choice of path.
- For example, in the figure, the work done by gravity when an object moves from B to C is $\mathrm{W(grav)_{BC}} = -mg(h - h_0)$.
- No work is done by gravity when the object moves horizontally from A to B,
  $\mathrm{W(grav)_{AB}} = -mg(h_0 - h_0) = 0$
- So, the total work done by gravity along the path ABC is $\mathrm{W(grav)}_{ABC} = -mg(h - h_0)$.
- When the block is moved vertically from A to C, the work done by gravity again is
  $\mathrm{W(grav)}_{AC} = -mg(h - h_0)$.
- Therefore, the work is the same for both paths.
- Any force that has the property that the work it does is the same for all paths between any two given points is said to be a conservative force.
- This property makes it meaningful to **associate a potential energy with a position**.
- Friction and many other forces are not conservative.



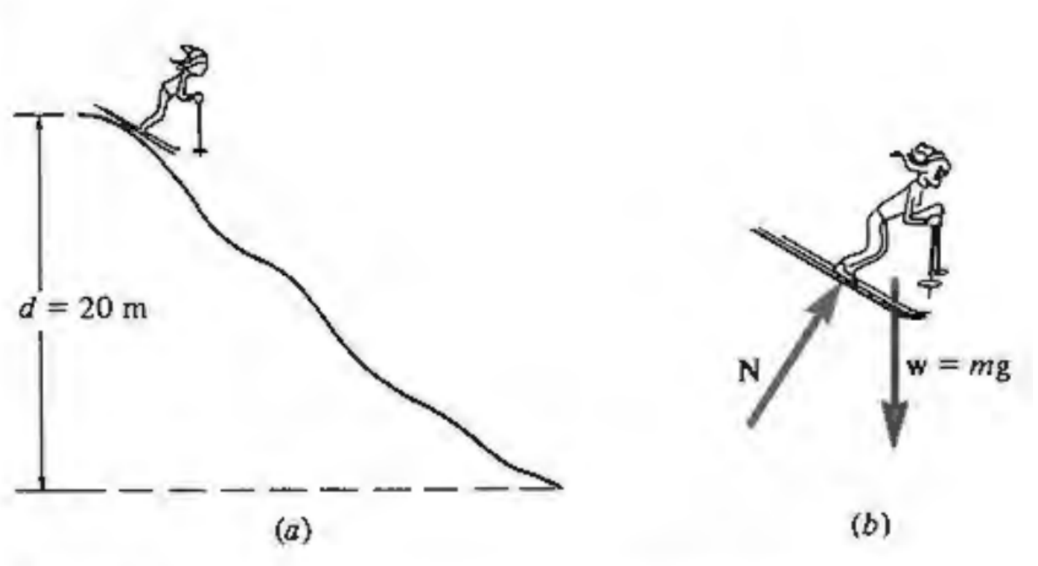
6.3 Potential energy:
Dissipative forces:

- Friction is not conservative because the work done by friction depends on the path.
- Friction always opposes the motion of an object, so it always does negative work.
- The energy expended by an object against the frictional forces is usually converted into thermal energy and hence is lost as mechanical energy
- Therefore, friction is referred to as a dissipative force.

6.3 Potential energy:

Example 6.6.

Suppose that as in the preceding example (example 6.5), a woman skis down a 20-m high hill. However, this time frictional forces are not negligible, so her speed at the bottom of the hill is only l0 m/s. How much work is done by frictional forces if her mass is 50 kg?



Answer:

choose the bottom of the hill as our reference level for calculating potential energies, which means her final potential energy is $\mathcal{U} = 0$. Also, her initial kinetic energy is still $K_0 = 0$.

$$E = E_0 + W_a$$

$$\tfrac{1}{2}mv^2 + 0 = 0 + mgd + W_a$$

$$\begin{aligned} W_a &= \tfrac{1}{2}mv^2 - mgd \\ &= \tfrac{1}{2}(50\text{ kg})(10\text{ m s}^{-1})^2 \\ &\quad - (50\text{ kg})(9.8\text{ m s}^{-2})(20\text{ m}) \\ &= -7300\text{ J} \end{aligned}$$

As anticipated, the work done by the applied force is negative, since frictional forces always oppose the motion. The skier has done 7300 J of work against friction, and this mechanical energy has been converted into thermal energy.

6.3 Potential energy:

Example 6.7.

skier reaches the flat ground at the bottom of the slope with a speed of 19.8 m s-r and then, by turning her skis sideways, quickly comes to a stop. If the coefficient of kinetic friction is 2.5, how far will she skid before coming to a halt?

Answer:

Since the ground is level, there is no potential energy change and all her kinetic energy, $\frac{1}{2}mv^2$, must be dissipated. The normal force is equal and opposite to her weight, so the work done by the frictional force over a distance s is

$$W_a = -\mu_k mgs$$

Thus

$$E = E_0 + W_a$$

$$0 = \tfrac{1}{2}mv^2 - \mu_k mgs$$

$$s = \frac{v^2}{2g\mu_k} = \frac{(19.8\ \text{m s}^{-1})^2}{2(9.8\ \text{m s}^{-2})(2.5)} = 8\ \text{m}$$

6.3 Potential energy:

Example

A block of mass 5 kg is moving on a rough surface. The coefficient of kinetic friction between the surface and the block is $\mu_k = 0.4$. If the block starts with initial velocity of 15 m/s, find the distance travelled at the instant its speed is 5 m/s?

Answer:

Since the ground is level, there is no change in the potential energy. The normal force is perpendicular to the displacement and so it does no work. The only work done is by the frictional force. The normal force is

$\mathrm{N} = mg$

The work done by friction is

$\mathrm{W_a} = f_k s\cos(180°) = -\mu_k N s = -\mu_k m g s$

$E = E_0 + W_a$

$K + \mathcal{U} = K_0 + \mathcal{U}_0 + W_a$

$\frac{1}{2} m v^2 + 0 = \frac{1}{2} m v_0^2 + 0 - \mu_k m g s$

$s = \frac{v_0^2 - v^2}{2(\mu_k g)} = \frac{15^2 - 5^2}{(2)(0.4)(9.8)} = 25.5\ m$

6.9 Power:

- When an amount of work ΔW is done in a time Δt, the average power is defined as the average rate of doing work

$$\overline{\mathcal{P}} = \frac{\Delta W}{\Delta t}$$

- The instantaneous power $\mathcal{P}$ is found by considering smaller and smaller time intervals, so

$$\mathcal{P} = \frac{dW}{dt}$$

- The instantaneous power can be written as

$$\mathcal{P} = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\mathbf{s}}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

- The S.I. power unit is a joule per second (J/s), which is called a watt (W).
- A unit of power in the U.S. customary system is the horsepower (hp): 1 hp=746 W.
- Energy is often sold by electrical utilities by the kilowatt hour ($\mathrm{kW\ h}$). **This is 1 kilowatt of power for 1 hour.** In terms of S.I. units,

$$1\ \mathrm{kW\ h} = (10^3\ \mathrm{W})(3600\ \mathrm{s}) = 3.6 \times 10^6\ \mathrm{J}$$

6.9 Power:

Example 6.14

A 70-kg man runs up a flight of stairs 3 m high in 2 s. (a) How much work does he do against gravitational forces? (b) What is his average power output?

Answer

(a) The work done, ΔW, is equal to his change in potential energy, mgh. Thus

$$\Delta W = mgh = (70 \text{ kg})(9.8 \text{ m s}^{-2})(3 \text{ m}) \\ = 2060 \text{ J}$$

(b) His average power is the work done divided by the time,

$$\overline{\mathcal{P}} = \frac{\Delta W}{\Delta t} = \frac{2060 \text{ J}}{2 \text{ s}} = 1030 \text{ W}$$

This is a high power output for a human.

6.9 Power:

Example 6.17

A 250-kg piano is raised by a hoist at a constant velocity of 0.1 m/s. What is the power expended by the hoist?

Answer

Since the force $\boldsymbol{F}$ is equal and opposite to the weight, $m\mathbf{g}$. So, it is parallel to the velocity,

$$\mathcal{P} = Fv = (250\ \text{kg})(9.8\ \text{m s}^{-1})(0.1\ \text{m s}^{-1})$$
$$= 245\ \text{W}$$