Chapter 5
Circular motion

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COURSE TOPICS:

- 5.1 Centripital accelration
- 5.2 Examples of circular motion

Introduction:

- Newton's laws of motion permit us to find the motion of any object if we know the forces acting upon the object and its initial position and velocity.

- Objects are in equilibrium and remain at rest if the net force is zero:
  $\boldsymbol{F}_{net} = 0$ then $\boldsymbol{a} = 0$ and $\boldsymbol{v} = 0$ or constant.

- Uniformly accelerated motion occurs when the net force is constant in magnitude and direction.
  $\boldsymbol{F}_{net} = \text{constant}$ then $\boldsymbol{a} = \text{constant}$

- In this chapter, we consider another kind of motion frequently encountered, **motion in a circular path**.

- When an object moves in a circular path at a constant speed, **its acceleration is directed toward the center of the circle**.

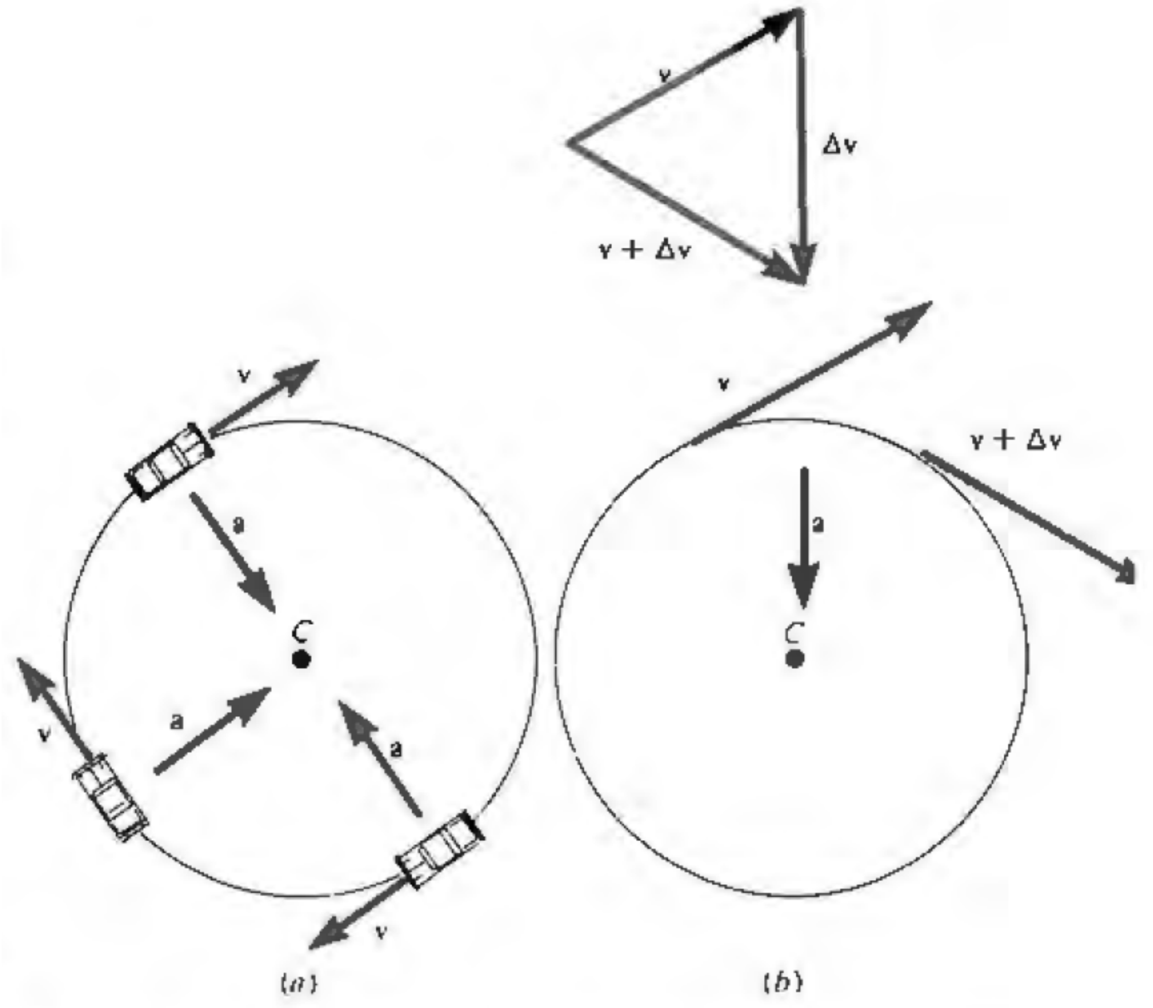
- The force required to produce this acceleration can be provided in various ways’
  - Friction provides the required force for a car on a flat, circular track;
  - gravity for an artificial satellite orbiting the earth;
  - and electrical forces for an electron orbiting an atomic nucleus

5.1 Centripetal acceleration:

- When an object is going around a circular track at a constant speed then it is said to be undergoing uniform circular motion,
  - It has no acceleration component along the direction of motion.
  - The velocity vector $\mathbf{v}$ is continually changing direction.
  - The velocity change $\Delta\mathbf{v}$ that occurs in a short time Δt points toward the center of the circle.
  - The acceleration $\boldsymbol{a} = \frac{\Delta \mathbf{v}}{\Delta \mathrm{t}}$ evaluated for a very small Δt,
  - so $\boldsymbol{a}$ also points toward the center of the circle or is directed radially inward.
  - For motion in a circle of radius r with speed v, this acceleration toward the center or centripetal acceleration has a magnitude of

$$\boxed{a_r = \frac{v^2}{r}}$$

The subscript r reminds us the acceleration is radial.



5.1 Centripetal acceleration:

Example 5.1

A car goes around a flat, circular track of radius 200 m at a constant speed of 30 m/s. What is its acceleration?

Answer

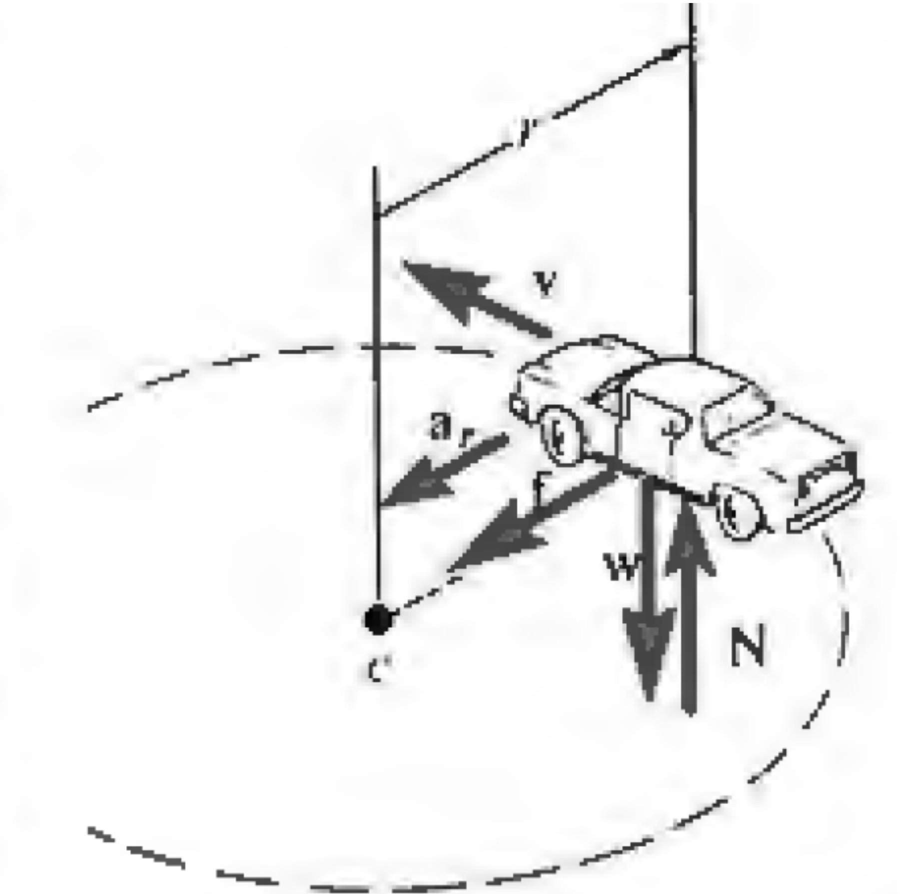
Since the speed is constant, there is no tangential acceleration. However, because the velocity vector is changing direction, there is an acceleration toward the center of the circle. This acceleration has a magnitude of

$$a_r = \frac{v^2}{r} = \frac{(30\ \text{m s}^{-1})^2}{200\ \text{m}} = 4.5\ \text{m s}^{-2}$$

5.1 Centripetal acceleration:

- If the object moves in a circular path, it has a radial acceleration $a_r = \frac{v^2}{r}$
  - Some force must be producing this acceleration.
  - From Newton's second law, the force must be equal to the mass times the acceleration.
  - Thus, the net force $\boldsymbol{F}$ needed to produce a centripetal acceleration a_r is

$$\boxed{F = ma_r = \frac{mv^2}{r}}$$

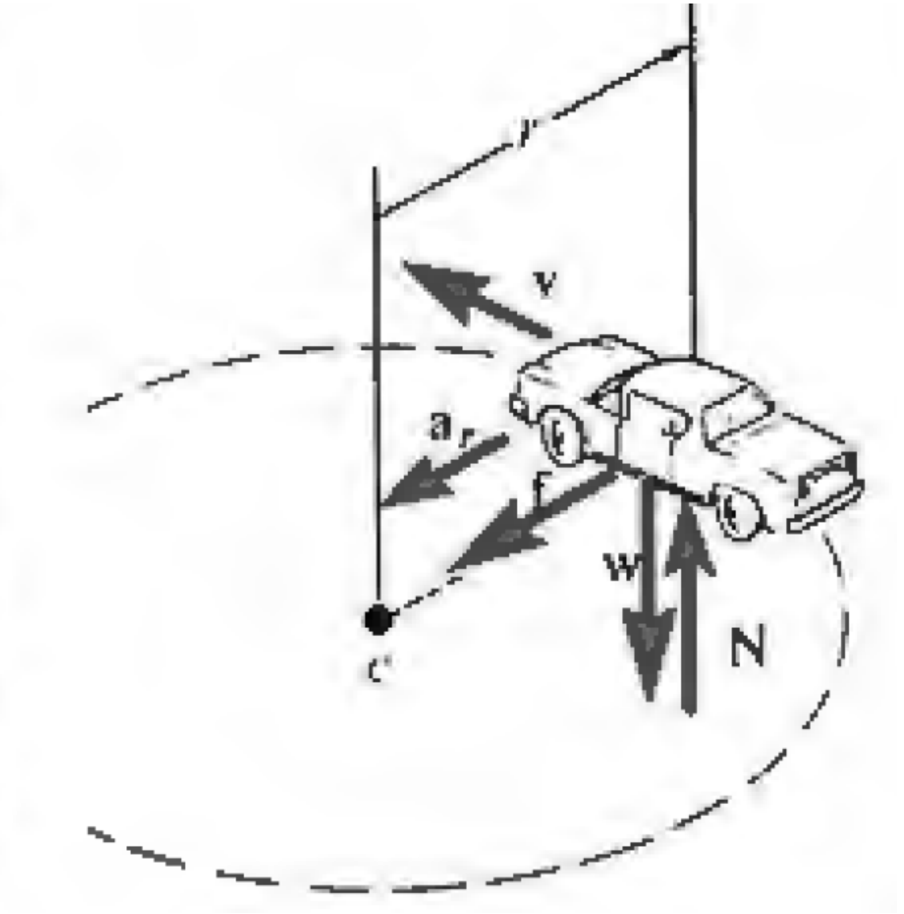


- Note that the central force is not a new kind of force. It might be caused by frictional, electric, magnetic or any kinf of forces.
- **Example**: In the case of a car on a flat circular track, the centripetal acceleration results from a frictional force exerted on the tires by the road
- Other forces also act on the car:
  - its weight, directed downward;
  - An equal but opposite upward normal force due to the road;
  - a backward force due to air resistance;
  - forward force exerted by the road on the tires.

5.2 Examples of circular motion:

Example 5.2

The car in the preceding example travels on a flat, circular track of radius 200 m at 30 m/s and has a centripetal acceleration $a_r = 4.5\ m/s^2$. (a) If the mass of the car is 1000 kg, what frictional force is required to provide the acceleration? (b) If the coefficient of static friction μ_s is 0.8, what is the maximum speed at which the car can circle the track?



Answer

(a) Since the mass is 1000 kg and the acceleration is $a_r = 4.5\ m/s^2$, then

$$F = ma_r = (1000\ \text{kg})(4.5\ \text{m s}^{-2}) = 4500\ \text{N}$$

(b) From the figure, the normal force N is equal in magnitude to the weight $w = mg$.

$$N = w = mg$$

Thus the maximum frictional force possible is

$$f_s = \mu_s N = \mu_s mg$$

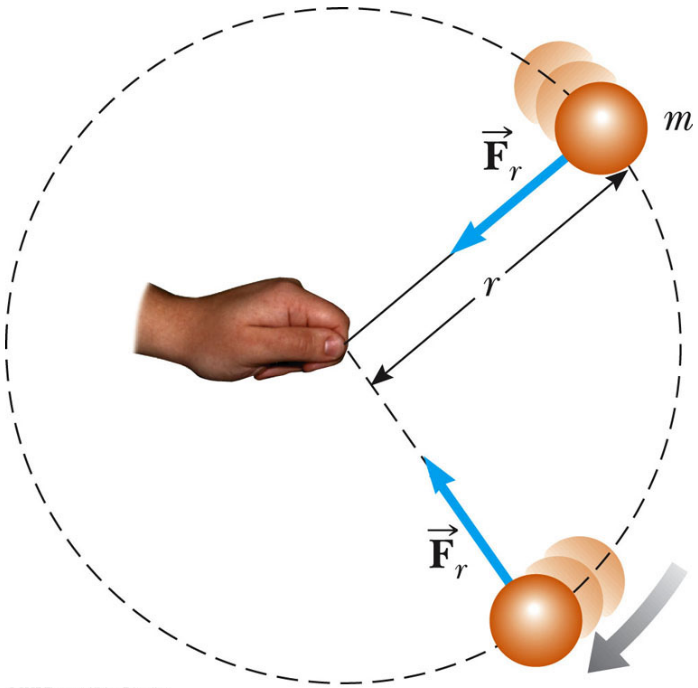
The maximum velocity satisfies

$$\frac{mv^2}{r} = f_s = \mu_s mg \quad \text{then} \quad v = \sqrt{\mu_s gr}$$

5.2 Examples of circular motion:

Example

A puck of mass 0.5 kg is attached to the end of a cord of length 1.5 m. The puck moves in a horizontal circle as shown. If the cord can withstand a maximum tension of 50 N. What is the maximum speed at which the puck can revolve before the cord breaks



Answer

(a) The tension in the cord is causing the centripetal acceleration. So,

$$F_r = T = \frac{mv^2}{r}$$

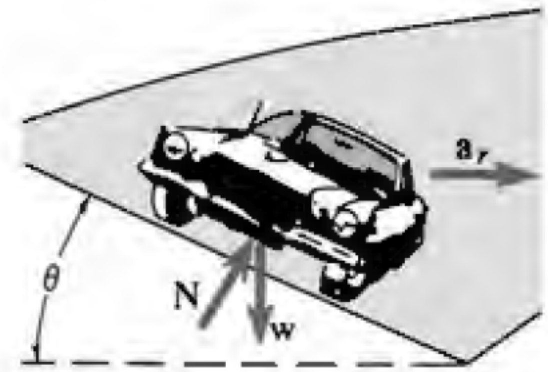
Then

$$v = \sqrt{\frac{Tr}{m}}$$

The maximum speed the puck can have corresponds to the maximum tension the cord can withstand

$$v_{\text{max}} = \sqrt{\frac{T_{\text{max}} r}{m}} = \sqrt{\frac{(50.0\ \text{N})(1.50\ \text{m})}{0.500\ \text{kg}}} = \boxed{12.2\ \text{m/s}}$$

5.2 Examples of circular motion:

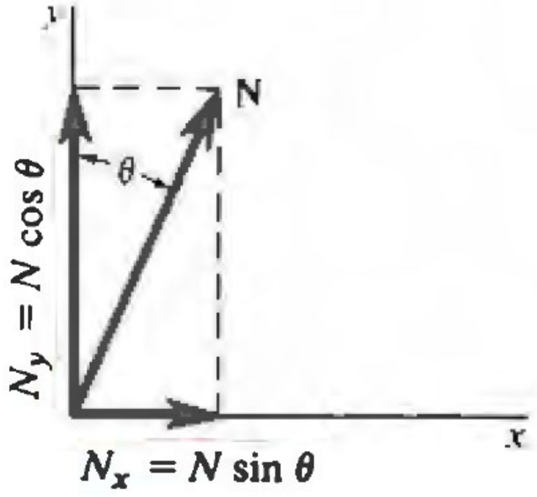


Good highways usually have banked or slanted curves, so that the normal force exerted by the road on the car has a horizontal component. This horizontal component can provide part of or all the force needed to produce the centripetal acceleration, reducing the role of the frictional force.

Example 5.3.

A curve of radius 900 m is banked, so no friction is required at a speed of 30 m/s. What is the banking angle θ?

Answer



The figure shows the relevant forces acting on the car: its weight $w = mg$ and the normal force $\boldsymbol{N}$. The forces along the direction of motion play no role here. Since the horizontal component of $\boldsymbol{N}$ must provide all the centripetal acceleration,

$$N \sin\theta = \frac{mv^2}{r}$$

There is no vertical acceleration component, so the net vertical force is zero

$$N \cos\theta = mg$$

Dividing the first equation by the second, we obtain

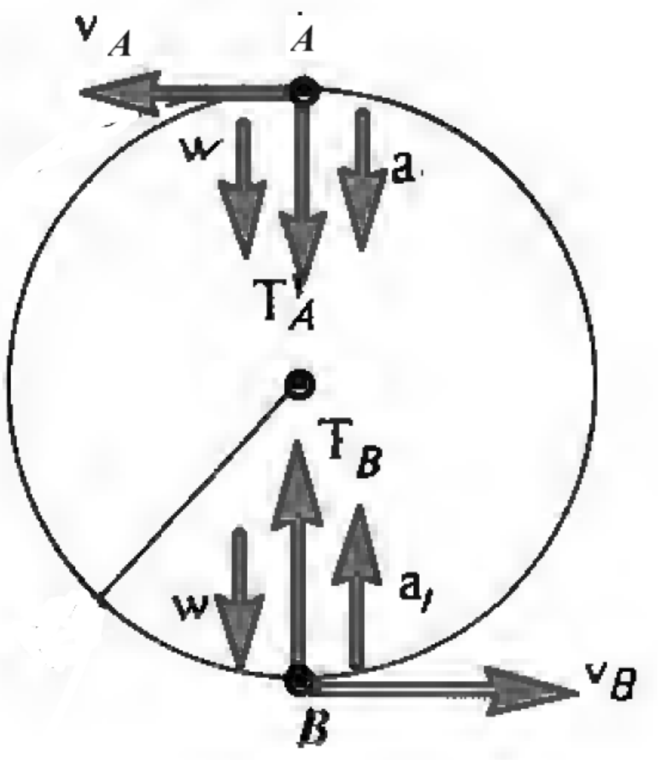
$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

Then

$$\tan\theta = \frac{v^2}{rg} = \frac{(30\ \mathrm{m\ s^{-1}})^2}{(900\ \mathrm{m})(9.8\ \mathrm{m\ s^{-2}})} = 0.102$$

$$\theta = 6^\circ$$

5.2 Examples of circular motion:



Example.

The figure shows a bucket of total mass m swinging in a circle of radius R. (a) find the tension in the cord at the top of the circle (point A), (b) Find the tension in the cord at the bottom of the circle (point B).

Answer

(a) At the top of the circle (point A), the tension $\boldsymbol{T}_A$ in the rope is parallel to the weight $\boldsymbol{w}$, so $\boldsymbol{F} = m\boldsymbol{a}$:

$$T_A + mg = \frac{mv_A^2}{R}, \qquad T_A = \frac{mv_A^2}{R} - mg$$

(b) At the bottom of the circle (point B), the tension $\boldsymbol{T}_B$ in the rope and the acceleration are opposite to the weight $\boldsymbol{w}$, so $\boldsymbol{F} = m\boldsymbol{a}$:

$$T_B - mg = \frac{mv_B^2}{R}, \qquad T_B = \frac{mv_B^2}{R} + mg$$

Physiological effects of circular motion:

- The whole human body is not rigid. The blood flows in vessels, so when the body accelerates upward the blood accumulates in the lower part and when accelerates downward the blood volume increases in the upper part of the body. Also, internal organs are not rigidly hold in place which cause unpleasant feeling.
- The ability of any person to withstand the acceleration depends on both the magnitude of the acceleration and its duration.
- Visual blackout occurs first at acceleration about $a \approx 3g \approx 30\ m/s^2$ because of the reduce of the blood pressure in the retina.
- The blood pressure drops because the heart has difficulties in pumping the blood with increased effective weight.
- Special training for pilots to reduce blood pooling in the lower body
- The reduced supply of blood to the brain leads to a complete blackout or unconsciousness at about $a \approx 6g \approx 60\ m/s^2$
- The human body detect the angular motion even if his eyes are closed. Not only angular motion but also motion direction, speed and stop can be detected by a sensor in the inner ear. The semicircle canals which contains a watery fluid called enddymph has a swimming door (cupola) which senses relative motion to fluid.