Chapter 4
Statics

Dr. Ghassan Alna'washi

COURSE TOPICS:

- 4.1 Torque
- 4.2 Equilibrium of rigid bodies
- 4.3 The center of gravity
- 4.4 Stability and balance
- 4.5 Levers and mechanical advantage

Introduction:

Statics is the study of the forces acting on an object (**rigid body)** that is in equilibrium and at rest.
rigid body: An object extended in space that does not change its size or shape when subject to a force

A rigid bod is will be in equilibrium if two conditions are satisfied:

1. The net force is zero is sufficient to ensure that a point particle at rest remains at rest:

$$\boldsymbol{F}_{\boldsymbol{net}} = \sum_{\boldsymbol{i}} \boldsymbol{F}_{\boldsymbol{i}} = \boldsymbol{F}_{\mathbf{1}} + \boldsymbol{F}_{\mathbf{2}} + \boldsymbol{F}_{\mathbf{3}} + \cdots = 0$$

2. The net turning effect (or net torque) is zero

$$\boldsymbol{\tau}_{\boldsymbol{net}} = \sum_{\boldsymbol{i}} \boldsymbol{\tau}_{\boldsymbol{i}} = \boldsymbol{\tau}_{\mathbf{1}} + \boldsymbol{\tau}_{\mathbf{2}} + \boldsymbol{\tau}_{\mathbf{3}} + \cdots = 0$$

The importance of Statics is to:

- Find the forces acting on various parts of engineering structures, such as bridges or buildings, or of biological structures, such as jaws, limbs, or backbones.
- Understand the force multiplication or mechanical advantage obtained with simple machines, such as the many levers found in the human body.

At the end of this chapter the student will be able to calculate the torque, the center of gravity of a rigid body, evaluate the mechanical advantage of a mechanical system and to apply these concepts to biological systems.

4.1 Torque

The cross (vector) product

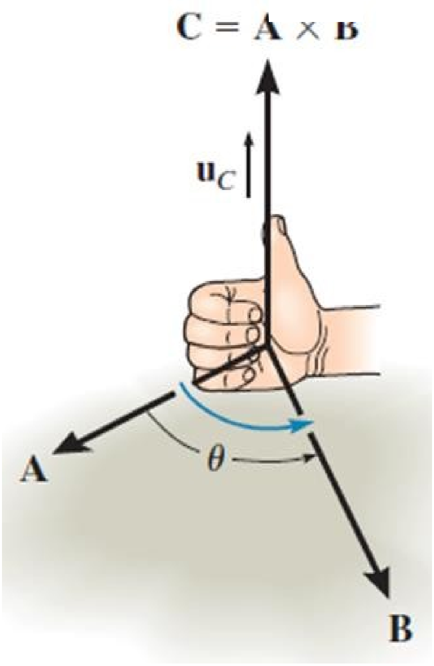
The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ which is written as :

$$\vec{C} = \vec{A} \times \vec{B}$$

The magnitude of the vector $\vec{C}$ is: $C = AB|\sin\theta|$

The direction of the vector $\vec{C}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ such that $\vec{C}$ is specified by *the right-hand rule*;

By curling the fingers of the right hand from vector $\vec{A}$ to vector $\vec{B}$, the thumb points in the direction of $\vec{C}$.



4.1 Torque

The cross (vector) product

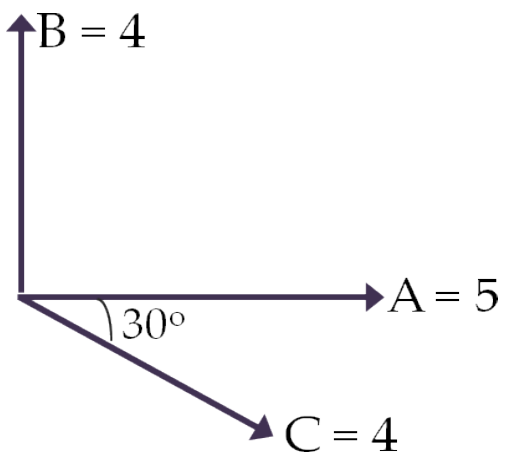
Example 4.2 The vectors in the figure are all in the plane of the page. Find

the magnitude and direction of :

(a) $\vec{A} \times \vec{A}$

(b) $\vec{A} \times \vec{B}$

c) $\vec{C} \times \vec{C}$.



Solution:

a) $|\vec{A} \times \vec{A}| = A.A.\sin(0°) = 0$

The cross product of any parallel vectors is zero

(b) The magnitude of $\vec{A} \times \vec{B}$ is:

$|\vec{A} \times \vec{B}| = A.B.\sin(90°) = 4 \times 5 \times 1 = 20$

We rotate our right palm from $\vec{A}$ toward $\vec{B}$. Our right thumb then points *out* of the page.

(c) The magnitude of $\vec{A} \times \vec{C}$ is:

$|\vec{A} \times \vec{C}| = A.C.\sin(30°) = 4 \times 5 \times 0.5 = 10$

Now when we rotate our right palm through 30° from $\vec{A}$ toward $\vec{C}$, our thumb points *into* the page.

4.1 Torque

In figure 4.1 a child applies equal but opposite forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ to opposite sides of a freely rotating seat. The seat will begin to rotate, hence, it is not in equilibrium even though the net force is zero. The rotation of the seat is due to the **Torque**.

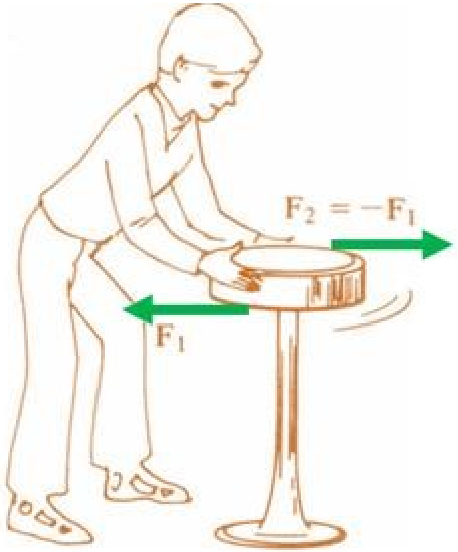
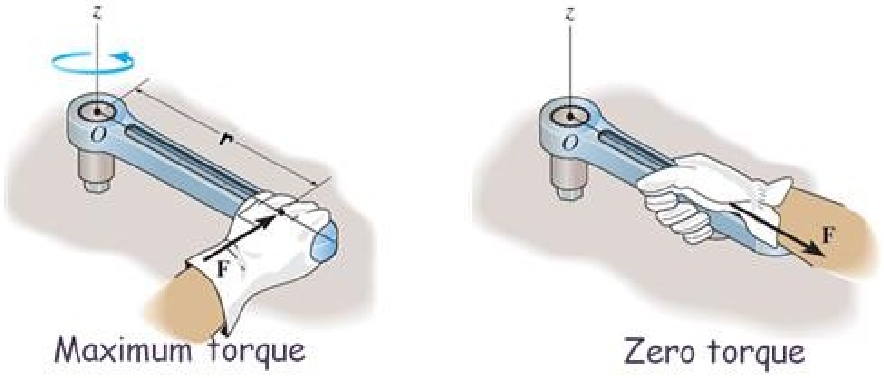


Figure 4.1

$$\vec{F}_1 + \vec{F}_2 = 0$$

Torque is the ability of a force to cause rotation



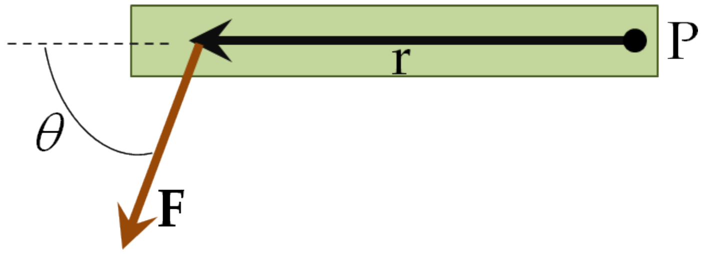
(a) Figure 4.2 (b)

Suppose we need to unscrew a large nut that rusted into place. To maximize the torque, we use the longest wrench available and exert as large a force as possible and pull at right angle. The torque is proportional to the magnitude of the force, to the distance from the axis of rotation and the direction of the force and to the angle between them.

4.1 Torque

If a rigid body is able to rotate about a point P, and a force $\boldsymbol{F}$ is applied on this rigid body, then the torque $\boldsymbol{\tau}$ about the point P is:

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$



The magnitude of the torque is

$$\tau = r\,F\,sin\theta$$

If $\theta = 0$ or π then $\tau = 0$

The direction of the torque is perpendicular to $\boldsymbol{r}$ and $\boldsymbol{F}$ and is determined by the right-hand rule

$\boldsymbol{r}$ is the distance from the pivot P to the point where the force acts on the object and θ is the angle from the direction of $\boldsymbol{r}$ to the direction of $\boldsymbol{F}$.

The SI unit of torque is the Newton meter ($N\,.m = kg.m^2/s^2$)

Example 4.1: A mechanic holds a wrench 0.3 m from the center of a nut. How large is the torque applied to the nut if he pulls at right angles to the wrench with a force of 200 N?

Solution:

Since he exerts the force at right angles to the wrench, the angle θ is 90°, and sin θ = 1 ,

Thus the torque is:

$$r = r\,F\,\sin\theta = 0.3\,m \times 200\,N \times 1 = 60\,N.m$$

4.1 Torque

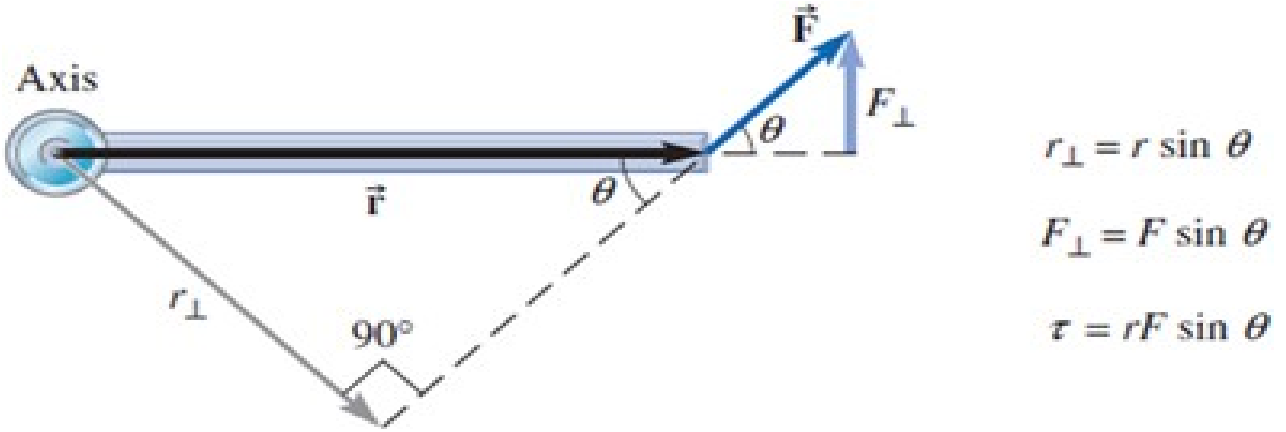
Finding the magnitude of a torque using the lever arm:

1. Draw a line parallel to the force through the force's point of application; this line (dashed in the figure) is called the force's line of action.
2. Draw a line from the rotation axis to the line of action. This line must be perpendicular to both the axis and the line of action. The distance from the axis to the line of action along this perpendicular line is the *lever arm* ($r_\perp$).

3. The magnitude of the torque is the magnitude of the force times the lever arm:

$r = r_\perp . F$ or $r = r . F_\perp$

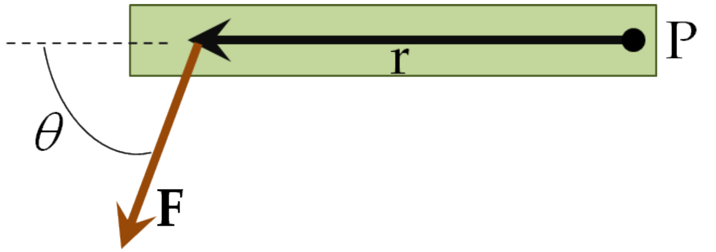
where $r_\perp = r \sin\theta$ and $F_\perp = F \sin\theta$



Example: A rigid body is able to rotate about a point P. Find the torque of a 50 N force acts at a point 2.5 m from point P at an angle of 75°.

Answer:

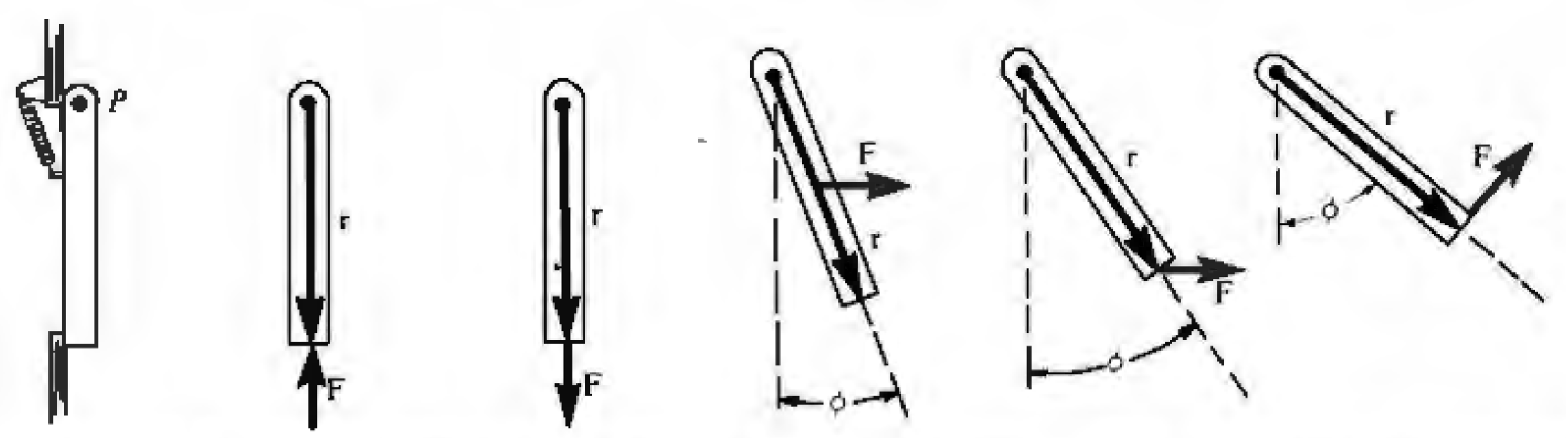
$\tau = rF sin\theta = (2.5\ m)(50\ N) sin 75° = 120.7\ N.m$



4.1 Torque

Example:

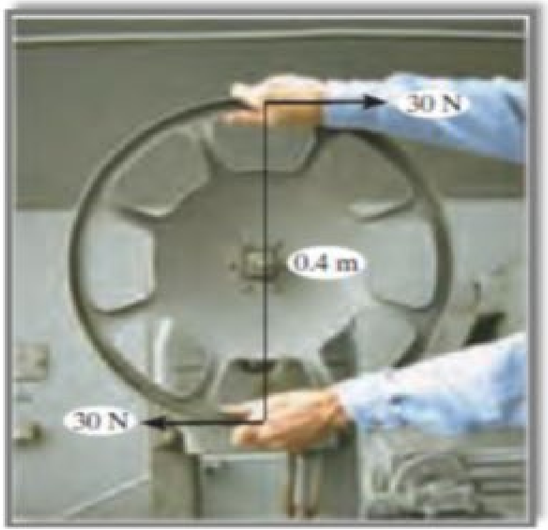
- The figure represents a top view of a screen door hinged at P.
- A spring resists opening of the door, so that the deflection angle ϕ increases with the applied torque.
- When a force $\boldsymbol{F}$ is applied opposite or parallel to $\boldsymbol{r}$, no torque results ($\theta = 0\ or\ \pi\ then\ \tau = rF\sin\theta = 0$).
- The greatest torque occurs when $\boldsymbol{F}$ is perpendicular to $\boldsymbol{r}$ and r is as large as possible.



4.1 Couples

A ***Couple*** is the total torque of a pair of forces with equal magnitudes but opposite directions acting along different lines of action.

- Couples do not exert a net force on an object even though they do exert a net torque.
- The net torque is independent of the choice of the point from which distances are measured.



Example 4.3 : Two forces with equal magnitudes but opposite directions act on an object with different lines of action (Fig 4.8). Find the net torque on the object resulting from these forces.

Solution: The torque resulting from the force at x_1 is $\tau_1 = x_1F$. The torque about P resulting from the force at x_2 is $\tau_2 = -x_2F$. (*The minus sign indicates a clockwise torque*). The net torque is: $\tau = \tau_1 + \tau_2 = x_1F - x_2F$

Then $\tau = (x_1 - x_2)F = -l\,F$

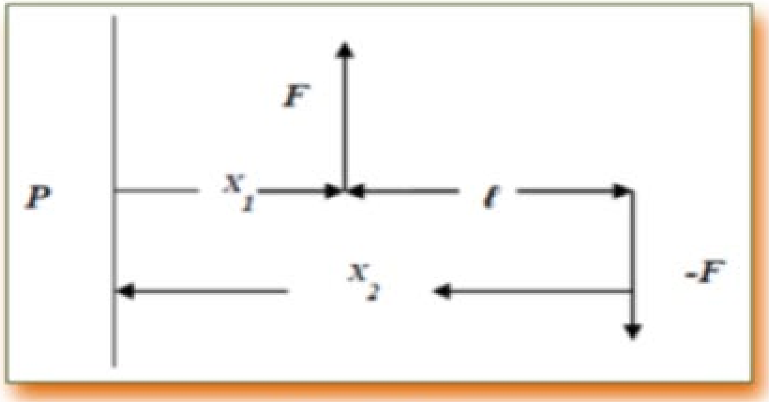


Fig. 4.8

The net torque is independent of the choice of the point from which distances are measured.

Note that the net force is zero ($\boldsymbol{F}_1 = -\boldsymbol{F}_2$) but the net torque is not zero (($\boldsymbol{\tau}_1 = -\boldsymbol{\tau}_2$) so the object is not in equilibrium

4.2 Equilibrium of rigid body

Two conditions for a rigid body to be in static equilibrium:

- *1. Translational equilibrium:* The net force on the rigid body must be zero: $\sum \vec{F} = 0$
- *2. Rotational equilibrium :* The net torque on the rigid body about any point must be zero:

  $\sum \vec{\tau} = 0$

Example 4.4 Two children of weights w_1 and w_2 are balanced on a board pivoted about its center.

- (a) What is the ratio of their distances x_2/x_1 from the pivot?
- (b) If $w_1 = 200\ N$, $w_2 = 400\ N$ and $x_1 = 1\ m$, what is x_2?

(For simplicity, we assume the board to be weightless; this will not affect the result.)

Solution :

a)The force N exerted by the support must balance the weights of the two children so that the net force is zero:

$$N = w_1 + w_2$$

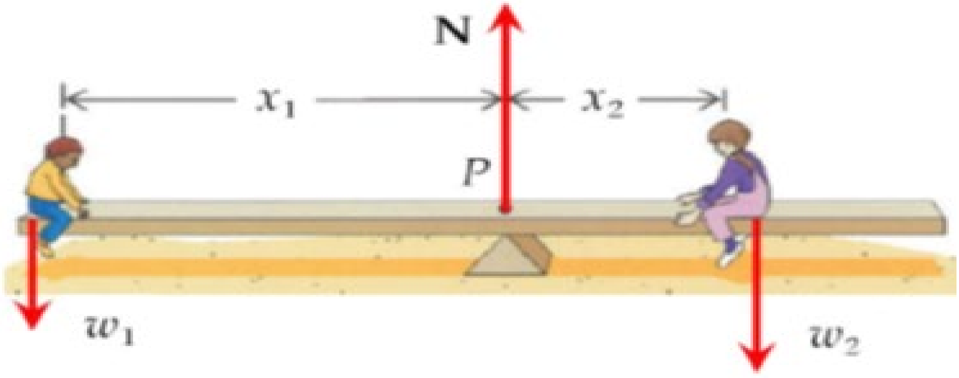
The torques resulting from the weights (about P) are:

$\tau_1 = x_1 w_1$ and $\tau_2 = -x_2 w_2$

Rotational equilibrium: $\tau = \tau_1 + \tau_2 = 0$

$$x_1 w_1 - x_2 w_2 = 0 \quad \text{or} \quad \frac{x_2}{x_1} = \frac{w_1}{w_2}$$

(b) $x_2 = x_1 \frac{w_1}{w_2} = (1\ m) \times \frac{200\ N}{400\ N} = 0.5\ m$



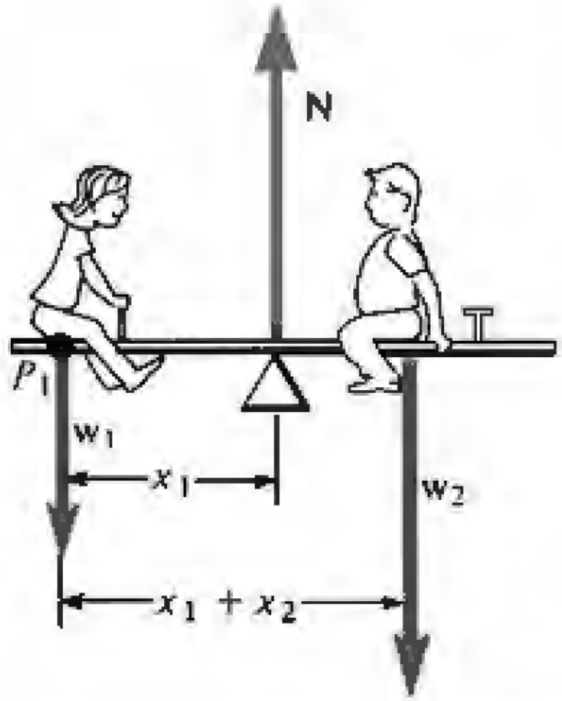
4.2 Equilibrium of rigid body

Example 2.5

Again, find x_2/x_1 for the seesaw of the preceding example, calculating torques about the point P_1, where the child of weight w_1 is seated.

Answer

To proceed, we redraw the force diagram as in the figure. Computing torques about P_1,



$$-(x_1 + x_2)w_2 + x_1 N = 0$$

But the forces must also add up to zero, so

$$N - w_1 - w_2 = 0$$

or

$$N = w_1 + w_2$$

Substitute in the torque equation

$$-(x_1 + x_2)w_2 + x_1(w_1 + w_2) = 0$$

Canceling some terms, we obtain our earlier result

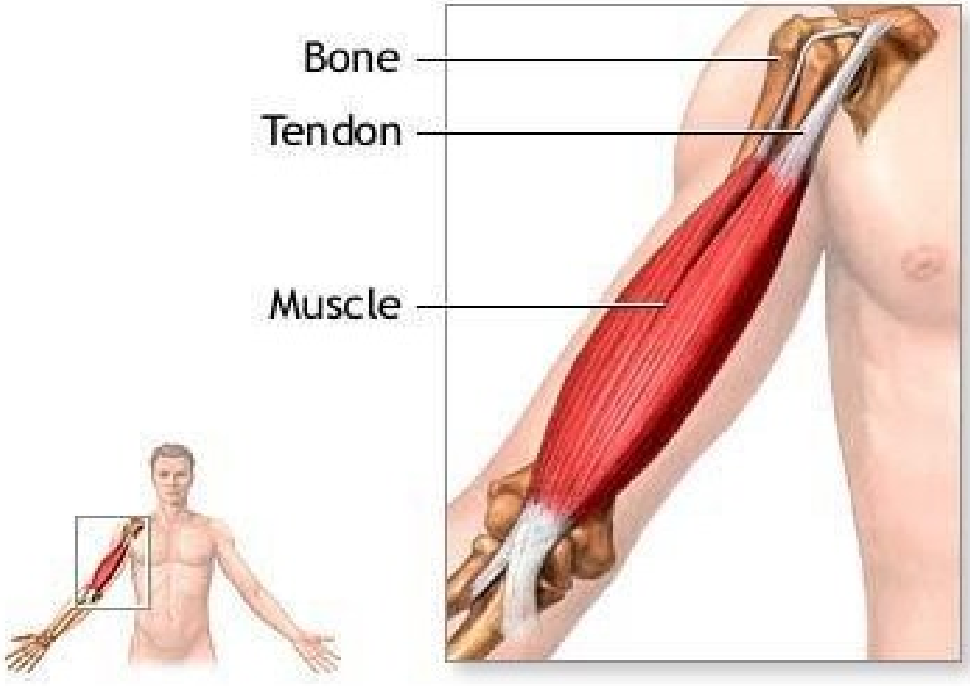
$$x_2 w_2 = x_1 w_1 \qquad \text{or} \qquad \frac{x_2}{x_1} = \frac{w_1}{w_2}$$

4.2 Equilibrium of rigid body

Applications to muscles and joints

The techniques for calculating forces and torques on bodies in equilibrium can be readily applied to the human body. This is of great use in studying the forces on *muscles*, *bones* and *joints*.

Generally a muscle is attached, via tendons, to two different bones . The points of attachment are called insertions. The two bones are flexibly connected at a joint, such as those at the elbow, knee and ankle. A muscle exerts a pull when its fiber contract under stimulation by a nerve, but it cannot exert a push.



4.2 Equilibrium of rigid body

Example 4.6 A model for the forearm in the position shown in the figure is a pivoted bar supported by a cable. The weight w of the forearm is $12\ N$ and can be treated as concentrated at the point shown. Find the tension T exerted by the biceps muscle and the force E exerted by the elbow joint.

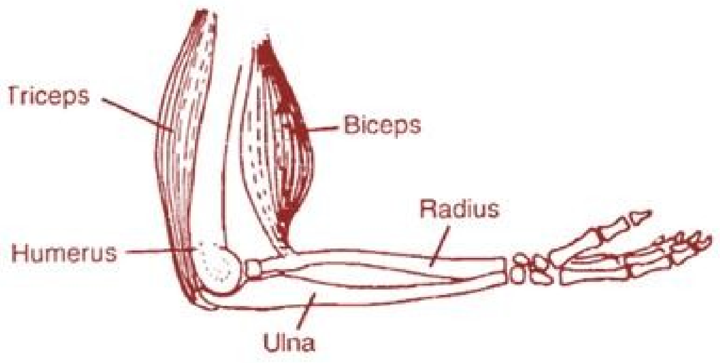


Fig. 4.11 a Forearm in horizontal position

Solution:

Applying the condition $\sum F = 0$, then $T - E - w = 0$

Both T and E are unknown.

Calculating torques about the pivot P:

E produces no torque.

w produces a torque: $\tau_w = -0.15 \times w$

T produces a torque: $\tau_T = 0.05 \times T$

At equilibrium the total torque is : $-0.15w + 0.05T = 0$

Then $T = \frac{0.15 \times w}{0.05} = \frac{0.15 \times 12}{0.05} = 36N$

Replacing in the first equation:

$E = T - w = 36\ N - 12\ N = 24\ N$

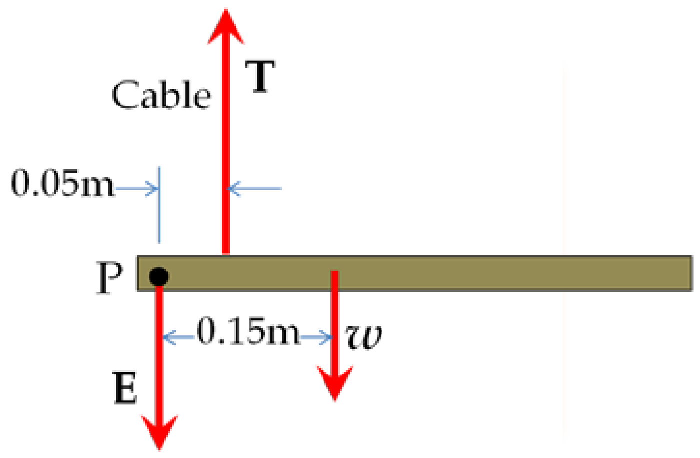


Fig. 4.12 a model of the forearm

4.2 Equilibrium of rigid body

Example 4.7.

An advertising sign is hung from a hinged beam that is supported by a cable. The sign has a weight $\mathrm{w} = 1000\ N$, and the weights of the beam and cable are negligible. Find the tension in the cable andthe force exerted by the hinge.

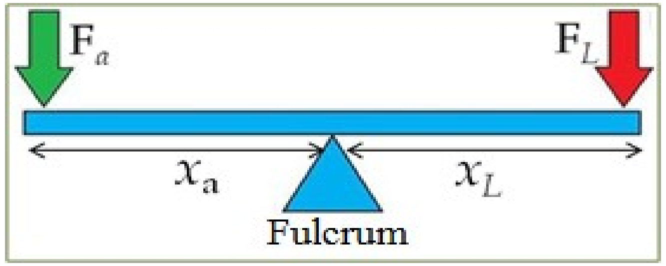
4.4 Levers and Mechanical Advantage

Simple machines, such as levers, pulleys are designed to reduce the force needed to lift a heavy load. In each case there is an applied force $\boldsymbol{F_a}$ and a load force $\boldsymbol{F_L}$.

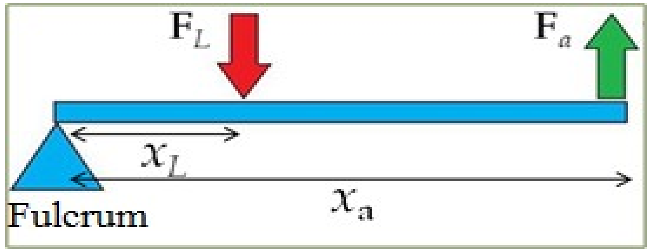
The mechanical advantage of a machine is the ratio of the magnitudes of the load force F_L balancedby an applied force F_a.

$$\textit{Mechanical Advantage:}\ M.A. = \frac{F_L}{F_a}$$

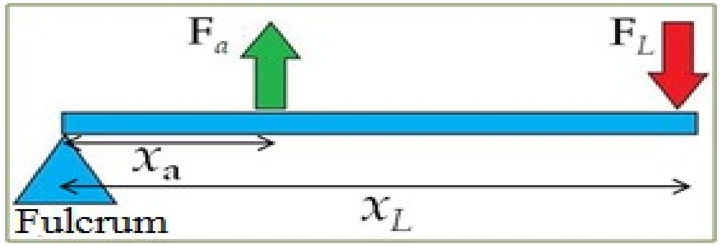
A *lever* in its simplest form is a rigid bar used with a fulcrum. *The fulcrum* is the point or support on which a lever pivots. Three classes of levers can be distinguished:



Class I : The fulcrum lies between F_a and F_L.



Class II: The fulcrum is at one end, F_a at the other end and F_L lies between F_a and the fulcrum.



Class III: The fulcrum is at one end, F_L at the other end and F_a lies between F_L and the fulcrum.

For all classes of levers the mechanical advantage can be expressed as a ratio of distances from the fulcrum. If the forces are at right angles to the lever, in equilibrium the ratio of the magnitudes of the load force to the applied force

$$\frac{F_L}{F_a} = \frac{x_a}{x_L} \quad \text{Hence,} \quad \text{M.A.} = \frac{x_a}{x_L} \quad (\text{forces} \perp \text{lever})$$

Example 4.10

Suppose the load F1 on a class I lever (Fie. 4.27a) has a magnitude of 2000 N. A person exerts a force $F_a = 500\ N$ to balance the load. (a) What is the ratio of the distances x_a and x_{L}

Answer

(a) To find x_a/x_{L}, we compute the torques about the fulcrum. For balance, the torques must sum to zero, so

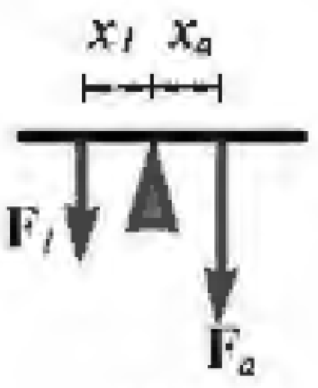
$$x_L F_L - x_a F_a = 0$$

and

$$\frac{x_a}{x_L} = \frac{F_L}{F_a} = \frac{2000\ \mathrm{N}}{500\ \mathrm{N}} = 4$$

(b) The mechanical advantage of the lever used in this way is then

$$\mathrm{M.A.} = \frac{F_L}{F_a} = \frac{x_a}{x_L} = 4$$



Examples of levers in the human body