# Chapter 3 Newton's laws of motion

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### **COURSE TOPICS:**

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- 3.2- Density
- 3.3- Newton's 1<sup>st</sup> law
- 3.4- Equilibrium
- 3.5- Newton's 3<sup>rd</sup> law
- 3.6- Newton's 2<sup>nd</sup> law
- 3.8- Some Examples of Newton's Laws
- 3.12- Friction



#### **Introduction:**

- Having learned how to describe motion, we can now turn to the more fundamental question of what causes motion. In a previous chapters the motion of an object was described without considering the causes of motion *(Forces)* and the object in motion was considered like a point particle without mass. In this chapter we introduce the concepts of mass and force.
- Although there are many kinds of forces in nature, their effects are described accurately by three general laws, first stated by Isaac Newton.
- Even though twentieth century advances have shown that **Newton's laws are inadequate** at the **atomic scale** and at velocities comparable to the **speed of light** (3x10<sup>8</sup> m/s). But these laws are fully adequate for must applications in different fields such as **astronomy, biomechanics, geology** and **engineering**.

# 3.1 Force, weight and gravitational mass

- *A force* is a vector quantity, it represents the ability to produce motion or to cause an object to change its state of motion.
- Two kinds of forces can be distinguished: field forces and contact forces
  - A field force is a force that acts through an empty space (gravitational force, electric force, magnetic force,...)
  - A contact force acts through a contact point or surface (pushing or pulling, friction force, tension on a string, reaction force,...)
- If more than one force act on an object, the <u>net force</u> or <u>resultant</u> is the sum of the individual forces:

$$\vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \dots = \sum \vec{F}$$

• The weight is the magnitude of the gravitational force exerted by the earth on an object of mass m:  $w = m \times g$ 

where g is the of gravity acceleration.

• S.I unit of force is Newton (N): 1 N=1 kg.m/s<sup>2</sup>



### 3.1 Force, weight and gravitational mass

- The mass can be thought of as a quantity of matter. But from a mechanical point of view, it can be seen as
  resistance to the motion. This resistance is also named INERTIA which is a measure of how is difficult to move
  or to change the state of motion of an object.
- But an object submitted to the gravitational force has the gravitational mass which is equal to its weight divided by the gravity acceleration: m = W/g
- For Example, a man who weighs 1000 N on the earth has a gravitational mass:

$$m = w / g = 1000 N / 9.80 m s^{-2} = 102 kg$$

• The same man on the moon has the weight :

$$w = m \times g_{moon} = 102kg \times 1.62 m/s^2 = 165 N$$

• In everyday usage, mass is sometimes referred to as "<u>weight</u>", the units of which may be pound or kilograms

## 3.2 Density

- The *density* of an object is an intrinsic property of a material, microscopically related to the atomic arrangement. At the macroscopic scale, the density is defined as the ratio of mass to the volume:
  - $\rho = \frac{m}{V}$
  - SI unit is  $kg/m^3$ .
  - Two different objects of the same size, made up from different materials, have different masses because they don't have the same density.

**Example 3.1 P 50:** A cylindrical rod of aluminum has a radius *R* = 1.2 *cm* 

and a length L = 2m. What is its mass?

**Solution:** The density of aluminum is  $\rho = 2700 \ kg/m^3$ .

The volume of a cylinder is  $V = \pi R^2 L$ .

Then its mass is :

 $m = \rho V = \rho \pi R^2 L = (2700 \ kg Tm^3) \times \pi \times (1.2 \times 10^{-2} m)^2 \times 2m = 2.44 \ kg$ 

Density of some substances				s .
substance		$ ho$ (kg $/m^3$ )	substance	$ ho(kg/m^3)$
Gold		19,300	Ice	917
Mercury		13,600	Olive Oil	920
Lead		11,300	Oxygen	1,43
Silver		10,500	Air	1,29
Iron		7860	Helium	0,179
Water (0°C)		1000	Blood (37°C)	1060

### **3.3 Newton's first law**

- □ Newton's first law states that:
  - Every object continues in a state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces acting upon it.
- An equivalent statement of the first law is that if there is no force on an object, or if there is no net force when two or more forces act on the object, then:

(1) an object at rest remains at rest, and

(2) an object in motion continues to move with constant velocity.

Newton's first law as stated does not hold true for someone who is accelerating.

 $v = 0 \text{ if the object was initially at rest} \\ F_{net} = 0 \text{ then} \qquad v = 0 \text{ if the object was initially in motion})$  v = constant (if the object was initially in motion)

## **3.4 Equilibrium**

- Newton's first law tells us that the state of an object remains unchanged whenever the net force on the object is zero (even though two or more forces act upon it). In this case the object is said to be in *equilibrium*.
- □ The first law applies to objects in *uniform motion* as well as to objects *at rest*.

There are three types of equilibrium: unstable, stable, and neutral

- **1 Unstable equilibrium**: a small displacement leads to an unbalanced force that further increase the displacement from the equilibrium location (ball in position A).
- **2 Stable equilibrium**: a small displacement leads to an unbalanced force that tends to restore the object to the equilibrium location (ball in position B).
- 3 Neutral equilibrium: it is in equilibrium at any location near C.



### 3.4 Equilibrium

**Example**: if the object is in equilibrium, then

 $F_{net} = 0$  $\mathbf{F}_2$  $F_1 + F_2 + F_3 + F_4 = 0$ **F**<sub>4</sub> Let's first find the components:  $F_{1x} = F_1, \qquad F_{1y} = 0$  $F_{2x} = 0$ ,  $F_{2y} = F_2$ θ  $F_{3x} = 0, \qquad F_{3y} = -F_3$  $F_{4x} = F_4 \cos(\pi - \theta) = -F_4 \cos \theta, \qquad F_{4y} = F_{4x} = F_4 \sin(\pi - \theta) = F_4 \sin \theta$ Thus  $F_{netx} = F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0 \quad \Longrightarrow \quad F_1 - F_4 \cos \theta = 0$  $\mathbf{F}_3$  $F_{nety} = F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0 \implies F_2 + F_4 \sin \theta - F_3 = 0$ 

## **3.3 Equilibrium**

**Example**: A woman stands motionless on the floor.

Since the woman is in equilibrium, the force *N* exerted by the floor on the woman is equal in magnituc w and opposite in direction:

 $F_{net} = 0$  $N + W = 0 \implies N - mg = 0 \implies N = mg$ 

**Example**: A sled moving with a constant velocity. Forces acting on it include the weight w. a normal force N, a frictional force f retarding its motion, and a force T exerted by the rope.

The vector sum of the forces is zero because the sled is in equilibrium.

 $\boldsymbol{T} + \mathbf{N} + \boldsymbol{W} + \boldsymbol{f} = 0$ 

**Example:** A traffic light suspended by two cables is in equilibrium under the action of three forces. The vector sum of the forces is zero.

 $\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{W} = \boldsymbol{0}$ 



## **3.4 Equilibrium**

**Example**: A book on the surface of a table is being pushed down by a force **F**.

The book is in equilibrium

 $\boldsymbol{F}_{net} = 0$ 

 $\boldsymbol{F} + \boldsymbol{N} + \boldsymbol{W} = \boldsymbol{0}$ 

The y-components are:

 $\implies \qquad N-F-mg=0 \qquad \implies \qquad N=F+mg$ 

Thus, the normal force is now always equal the gravitational force.



## **3.5 Newton's third law:**

- If one object exerts a force F on a second, then the second object exerts an equal but opposite force -F on the first.
- For example: suppose you are at rest in a swimming pool. If you push a wall with your legs, the wall exert a force that propels you further into the pool. The *reaction* force the wall exerts on you is opposite in direction to the *action* force you exert on the pool.

#### Example 3. 2 P 52:

A woman has a mass of 60 kg. She is standing on a floor and remains at rest. Find the normal force exerted on her by the floor. **Solution:** 

The woman is in equilibrium, then the net force on the woman must be Zero:  $w + F_N = 0$ . The normal force is the force exerted by the floor on the woman. This force must have the same magnitude as her weight, which acts downward. Symbolically:

Its magnitude is  $F_N = mg = 60 \ kg \times (9.8 \ m/s^2) = 588 \ N.$ 

Note that the gravitational force and the normal force are not an action and a reaction forces because they are applied on the same object.





# 3.6 Newton's second law:

• The second law states that:

Whenever there is a net force acting on an object, this object will undergoes an <u>acceleration</u> in the same direction as the force.

- The acceleration is proportional to the net force and inversely proportional to the mass.
- Thus, we can relate the net force *F* and the acceleration *a* by Newton's second law:

#### F = ma

- The proportionality constant *m* is the *inertial mass* of the object.
- Note that if the net force is equal zero, then, the acceleration is also zero, which means that the velocity is constant or zero.
- The second law is consistent with the first law

- The systematic procedure for relating the acceleration of an object or objects to the forces present are:
  - 1. For each object, we draw a careful sketch showing the forces acting on that object.
  - 2. We then apply Newton's second law F = ma to each object separately. If there are *n* forces  $F_1$ ,  $F_2$ , ...,  $F_n$  acting on an object, the net force *F* is the sum of the forces, and we have

 $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n = m\mathbf{a}$ 

in components form, this is

$$F_x = F_{1x} + F_{2x} + \cdots + F_{nx} = ma_x$$
  
 $F_y = F_{1y} + F_{2y} + \cdots + F_{ny} = ma_y$ 

- 3. solve the equations for the unknown quantities symbolically and then substitute the numbers if they are given.
- 4. include the units of the numerical quantities and see that the final answer has the correct dimensions.

**Example 3. 6 P55:** A child pushes a sled across a frozen pond with a horizontal force of 20 *N*. Assume friction is negligible.

a) If the sled accelerates at  $0.5 m s^{-2}$ , what is its mass?

b) Another child with a mass of  $60 \ kg$  sits on the sled. What

acceleration, the same force produces now?

#### Solution of 3. 6 P55:

a) The gravitational force and the normal force cancel each other. Thus the net force exerted on the sled is a horizontal force with a magnitude of F = 20N. According to the second law:

$$m = \frac{F}{a} = \frac{20 N}{0.5 m s^{-2}} = 40 kg$$

b) The total mass of the sled becomes: m = 40 kg + 60 kg = 100 kgThen the acceleration is:

$$a = \frac{F}{m} = \frac{20 N}{100 \, kg} = 0.2 \, ms^{-2}$$



**Example 3.7:** An elecvator has a mass of 1000 kg. (a) It accelerates upward at  $3 m/s^2$ . What is the force T exerted by the cable on the elevator? (b) What is the force T if the acceleration is  $3 m/s^2$  downward? (c) What is the force T if the elevator moves with constant velocity

#### Answer

(a) The forces on the elevator are its weight *w* and the upward force *T* resulting from the cable. Apply Newton's second law:

 $F_{1y} + F_{2y} = ma_y$ With  $F_{1y} = T$  and  $F_{2y} = -w = -mg$ , this becomes  $T - mg = ma_y$  $T = m(g + a_y)$ Then, with  $a_y = 3 \text{ m s}^{-2}$  and m = 1000 kg,  $T = (1000 \text{ kg})(9.8 \text{ m s}^{-2} + 3 \text{ m s}^{-2})$ = 12,800 N





(b) The above equation  $T = m(g + a_y)$  holds true no matter what the magnitude or sign of  $a_y$ . When **a** is directed downward,  $a_y = -3 m/s^2$ , then

$$T = m(g + a_y)$$
  
= (1000 kg)(9.8 m s<sup>-2</sup> - 3 m s<sup>-2</sup>) = 6800 N

(c) In the case the elevator moves with constant velocity we have  $a_y = 0$ .

$$T = m(g + a_y) = (1000kg)(9.8m s^{-1} + 0) = 9800 N$$

Example 3.8: An ice hockey player strikes a puck of mass 0.17 kg with his stick, accelerating it along the ice from rest to a speed of  $20 m/s^2$  over a distance of 0.5 m. What force must he exert if the frictional force between the puck and the ice is negligible? (Assume the acceleration is constant.) Answer

We first find the acceleration and then apply Newton's second law. Using the uniform acceleration relation with  $v_i = 0$ ,  $v_f = 20 m/s$  and  $\Delta x = 0.5 m$ 

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{(20^2 - 0)m^2/s^2}{2 \times 0.5 m} = 400 \ m/s^2$$

Then the net force on the puck is

 $F = ma = (0.17 \text{ kg})(400 \text{ m s}^{-2}) = 68 \text{ N}$ 





**Example 3.9:** A child pulls a train of two cars with a horizontal force **F** of 10 N. Car 1 has a mass of  $m_1 = 3 kg$ , and car 2 has a mass  $m_1 = 1 kg$ . The mass of the string connecting the cars is small enough so it can be set equal to zero, and friction can be neglected. (a) Find the normal forces exerted on each car by the floor. (b) What is the tension in the string? (c) What is the acceleration of the train? Answer



$$w_1 = w_1 = 0$$

$$N_1 = w_1 = m_1 g = (3 \text{ kg})(9.8 \text{ m s}^{-2}) = 29.4 \text{ N}$$





(b) Both cars accelerate with the same acceleration a. Apply Newtons second law for the x-motion

for car 1:  $F_x = m_1 a_x$  $F - T = m_1 a_x$ 

for car 2:

$$F_x = m_2 a_x$$
$$T = m_2 a$$

Solving for *T*, we get:

$$T = \frac{F}{1 + \frac{m_1}{m_2}} = \frac{10 N}{1 + \frac{3 kg}{1 kg}} = 2.5 N$$

(c) Now we can find *a*:

$$T = m_2 a$$
  
 $a = \frac{T}{m_2} = \frac{2.5 \text{ N}}{1 \text{ kg}} = 2.5 \text{ m s}^{-2}$ 

**Example 3.10:** A block of mass  $m_1 = 20 kg$  is free to move on a horizontal surface. A rope, which passes over a pulley, attaches it to a hanging block of mass  $m_2 = 10 kg$ . Assuming for simplicity that the pulle and rope masses are negligible and that there is no friction, find (a) the forces on the blocks and (b) their acceleration (c) If the system is initially at rest, how far has it moved after 2 *s*?**Answer** 

(a) We first apply F = ma to  $m_1$ . Since it has no vertical acceleration component, then

$$N_1 = w_1 = m_1 g = (20 \text{ kg})(9.8 \text{ m s}^{-2}) = 196 \text{ N}$$

The system is accelerating with an unknown acceleration *a*:

$$F_x = m_1 a_x$$
$$T = m_1 a$$

Apply Newton's second law to block 2.Since it is accelerating downward,  $a_y = -a$ ,

$$F_{y} = ma_{y}$$
$$T - w_{2} = -m_{2}a$$



$$T = \frac{w_2}{1 + \frac{m_2}{m_1}} = \frac{m_2 g}{1 + \frac{m_2}{m_1}} = \frac{(10 \text{ kg})(9.8 \text{ m s}^{-2})}{1 + \frac{10 \text{ kg}}{20 \text{ kg}}}$$
  
= 65.3 N

Using the equation:  $T = m_1 a$ the acceleration is

(b) Solve for T and a, we get

$$a = \frac{T}{m_1} = \frac{65.3 \text{ N}}{20 \text{ kg}} = 3.27 \text{ m s}^{-2}$$

(c) Since the system is initially at rest and uniformly accelerated, the distance it moves in 2 s is

$$\Delta x = \frac{1}{2}a \ (\Delta t)^2 = \frac{1}{2}(3.27 \text{ m s}^{-2})(2 \text{ s})^2 = 6.54 \text{ m}$$

#### **3.12 Friction:**

*Friction is a force* that always *acts to resist the motion* of one object on another. Frictional forces are very important, since they make it possible for us to walk, use wheeled vehicles,....

Frictional forces can exist between solid surfaces or between fluids (**viscous forces**). Viscous forces are quite small compared to the friction between solid surfaces. Thus the use of lubricating liquid such as oil, which clings to the surface of metals, greatly reduces friction.

When we walk or run, we are not conscious of any friction in our knees or other leg joints. These joints, really, are lubricated by *synovial fluid*, which is squeezed through the cartilage lining the joints when they move. This lubricant tends to be absorbed when the joint is stationary, increasing the friction and making it easier to maintain a fixed position.



# **3.12 Friction: Static friction**

We distinguish two types of friction:

- Static friction, between two surfaces at rest.
- Kinetic friction, between two surfaces one moves against the other.

#### **Static friction**

We consider a block at rest on a horizontal surface (Fig. 3.19).

Since the block is at rest Fig. 3.19 a, the first law requires that the net force on the block be zero.

The vertical forces are the weight w and the normal force N, so we must have : N = w.



# **3.12 Friction: Static friction**

If there is no force applied in the horizontal direction and the block remains at rest (Fig 3.19 b),

the frictional force must be zero, according to the first law.

If we apply a small horizontal force T to the rights and if the block remains at rest (Fig. 3.19 c), the friction force  $f_s$  can no longer be zero, since the first law requires that the net force on the block be zero, then a frictional force opposite to the applied force must be appeared ( $f_s = -T$ )

If T is gradually increased  $f_s$  increases also. Eventually when T become large enough, the block begins to slide.

#### The static friction force attains a maximum value called the maximum static friction $f_{smax}$

Experimentally it is found that  $f_{smax}$  has the following properties:

- 1  $f_{smax}$  is independent of the contact area.
- 2 For a given pair of surfaces  $f_{smax}$  is proportional to the normal force **N**:

$$f_{smax} = \mu_s N$$

# **3.12 Friction:** Static friction

 $\mu_s$  is the coefficient of static friction and N is the magnitude of the normal force.

The coefficient of static friction  $\mu_s$  depends on:

- The nature of the two surfaces in contact
- Their cleanliness and smoothness
- The amount of moisture present

For metal on metal:  $\mu_s$  is between 0.3 and 1. When lubricating oils are used  $\mu_s$  is about 0.1. For

Teflon on metals,  $\mu_s \cong 0.04$ 

#### 3.12 Friction: Kineticfriction

 The force necessary to keep an object sliding at constant velocity is smaller than that

required to start it moving.

Thus the sliding or kinetic friction  $f_k$  is less than  $f_{smax}$ .

• The kinetic friction is independent of the contact area, it satisfies:



- Here μ<sub>k</sub> *is the coefficient of kinetic friction* and is determined by nature of the twosurfaces.
- $\mu_k$  is nearly independent of the velocity (since  $f_k < f_{smax}$  then  $\mu_k < \mu_s$ )

PREPARATORY YEAR

## 2.12 Friction

#### Example 3.17 p 149:

A 50 -*N* block is on a flat, horizontal surface. (a) If a horizontal force T = 20 N is applied and the block remains at rest ; what is the frictional force? (b) the block starts to slide when *T* is increased to 40 *N*. What is  $\mu_s$ ? (c) the block continues to move at constant velocity if *T* is reduced to 32 *N*. What is  $\mu_k$ ?

#### Solution:

a) Since the block remains at rest 
$$f_s = T = 20 N$$

b) 
$$f_{smax} = 40 N = \mu_s F_N$$
 then  $\mu_s = \frac{f_{smax}}{F_N} = \frac{40 N}{50 N} = 0.8$ 

c) When the block is moving at constant velocity with  $f_k = \mu_k F_N$ 

Then  $\mu_k = \frac{f_k}{F_N} = \frac{32 N}{50 N} = 0.64$ 

### 3.12 Friction

#### Example: Repeat example 3.9 with friction

A child pulls a train of two cars with a horizontal force  $\mathbf{F}$  of 10 N. Car 1 has a mass of  $m_1 = 3 kg$ , and car 2 has a mass  $m_1 = 1 kg$ . The mass of the string connecting the cars is small enough so it can be set equal to zero, and a constant friction force of  $f_k = 2 N$  acts on each car. (a) Find the normal forces exerted on each car by the floor. (b) What are the acceleration and the tension in the string **Answer** 

(a) Since 
$$a_y = 0$$
 for each car, then  
 $F_y = ma_y = 0$   
 $N_2 - w_2 = 0$   
 $N_2 = w_2 = m_2g = (1 \text{ kg})(9.8 \text{ m s}^{-2}) = 9.80 \text{ N}$   
and

 $N_1 - w_1 = 0$ 

$$N_1 = w_1 = m_1 g = (3 \text{ kg})(9.8 \text{ m s}^{-2}) = 29.4 \text{ N}$$





#### **3.12 Friction**

(b) Both cars accelerate with the same acceleration a. Apply Newtons second law for the x-motion

for car 1:

$$F_x = m_1 a_x$$
  
$$F - f_k - T = m_1 a$$

for car 2:

$$F_x = m_2 a_x$$
$$T - f_k = m_2 a$$

Solving for *a*, we get:

$$a = \frac{F - 2f_k}{m_1 + m_2} = 1.5 \ m/s^2$$

Now we can find *T* from any equation:

$$T - f_k = m_2 a$$
  
$$T = f_k + m_2 a = 3.5 N$$

## **3.9 Gravitational forces**

- The gravitational force between two masses is given by the law of universal gravitation.
- The taw of universal gravitation states that all objects in the universe attract each other.
- If two spheres or particles have gravitational masses *m* and *m*', and their centers are separated by a distance *r*, the forces between the two spheres have a magnitude

$$F = \frac{Gmm'}{r^2}$$



• G is called the gravitational constant and has a measured value of

 $G = 6.67 \times 10^{-11} N \cdot m^2 / kg^2$ 

- The gravitational forces are directed along the line connecting the centers of the two spheres.
- The magnitude of the gravitational force varies as  $1/r^2$  is referred to as an **inverse square law**.

#### **3.9 Gravitational forces**

**Example 3.12** The centers of two 10 kg spheres are separated by 0.1 m. (a) What is their gravitational attraction? (b) What is the ratio of this attraction to the weight of one of the spheres?

Answer: (a) Using Newton's law of gravitation, the forces between the spheres have a magnitude

$$F = G \frac{mm'}{r^2} = (6,67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}) \frac{(10 \text{ kg})(10 \text{ kg})}{(0,1 \text{ m})^2}$$
$$= 6,67 \times 10^{-7} \text{ N}$$

(b) The weight of one of the spheres is

$$w = mg = (10 \text{ kg})(9.8 \text{ m s}^{-2}) = 98 \text{ N}$$

Thus, the ratio of the gravitational forces between the spheres to the weight of a sphere is

$$\frac{F}{w} = \frac{6.67 \times 10^{-7} \text{ N}}{98 \text{ N}} = 6.81 \times 10^{-9}$$

Chapter three: Newton's Laws of monon ① find the net force on the blocks in the following cases 1 ION (magnetude and direction) (a) V20N 53° 20 N (c) (b) (d) (2) Hydrogen atoms are the most common types of matter in space (stime in the second of the second o if the mass of the H atom = 1.67×10<sup>-27</sup> kg and there is a 1 atom/cm<sup>3</sup> what is the density of the hydrogen in S. I units. (kg/m<sup>3</sup>)