Chapter 2
Motion in two dimensions

Dr. Ghassan Alna'washi

COURSE TOPICS:

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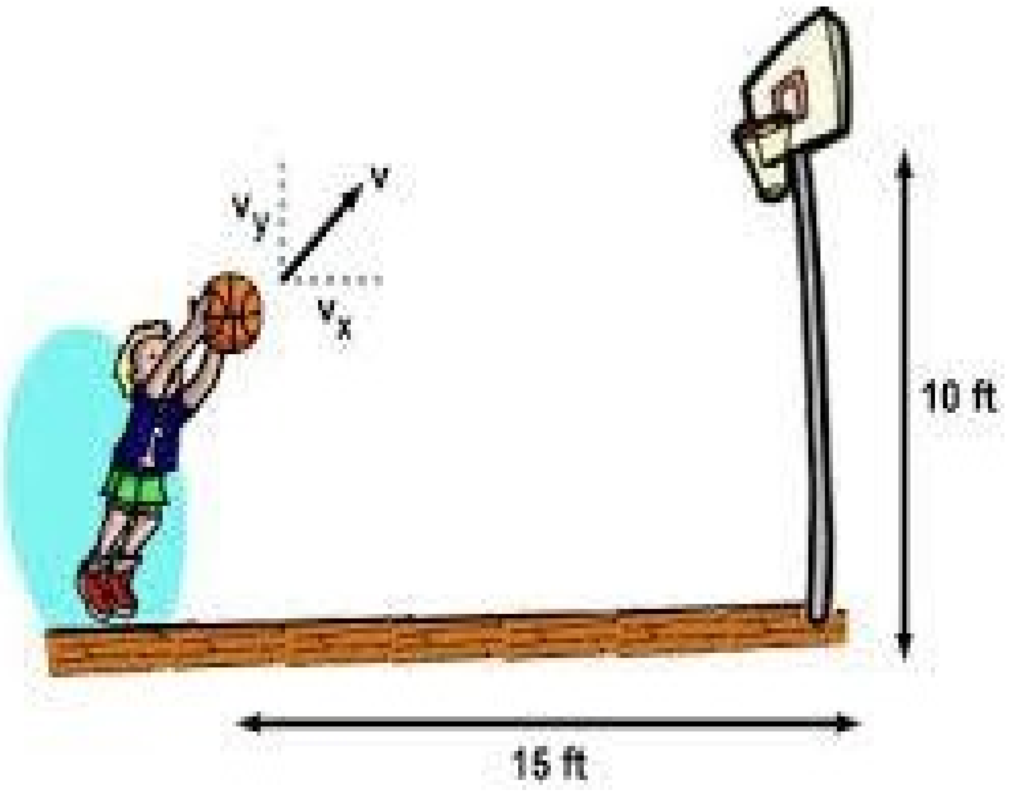
- An introduction to vectors
- The velocity in two dimensions
- The acceleration in two dimensions

Examples to be explained and solved: 2.1, 2.2, 2.3,

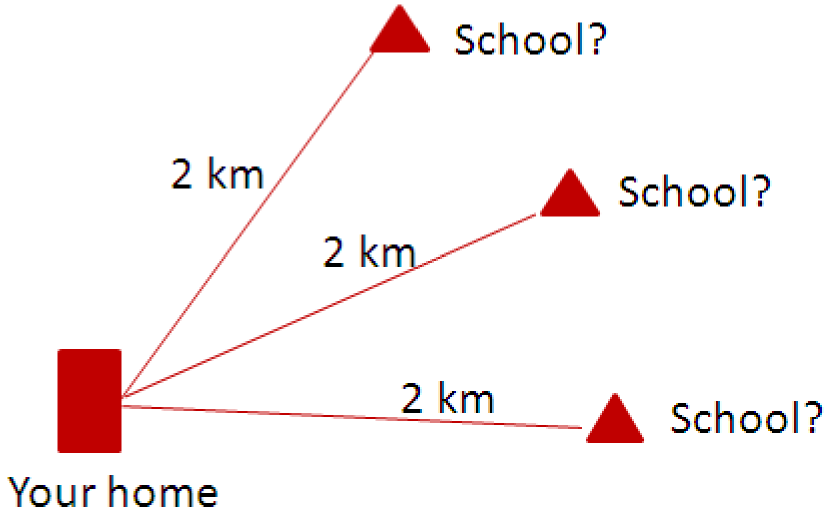
2.4 and 2.6



2.1 An introduction to vectors

- Physical quantities can be classified as ***scalars* or *vectors*.**
  - A **scalar quantity** is simple number (with unit) such as *mass, distance, speed*
  - a **vector quantity** is defined with both a magnitude (which is a number with unit) and a direction; such as *force, displacement, velocity*,….

For example, to describe where the school is located versus your home, it is not sufficient to give the distance between them. We have to give the **distance** and the **direction**.



Along this chapter we will see how to describe motion in two dimensions using vectors. That's why it's first important to know what is a vector and how to add, subtract and multiply vectors.

2.1 An introduction to vectors

Properties of vectors:

- Two vectors are ***equal*** if they have the same magnitude and the same direction.
  - $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ if $|\mathbf{A}| = |\mathbf{B}|$ and they point along parallel lines

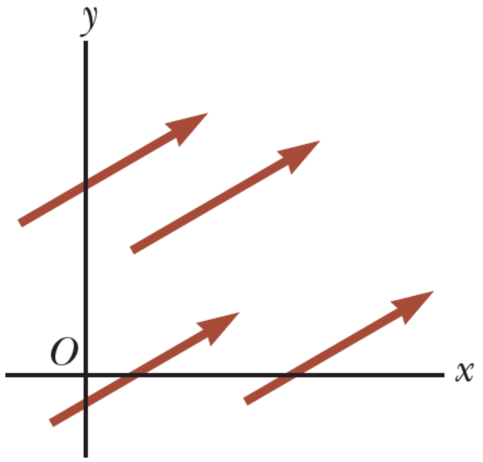
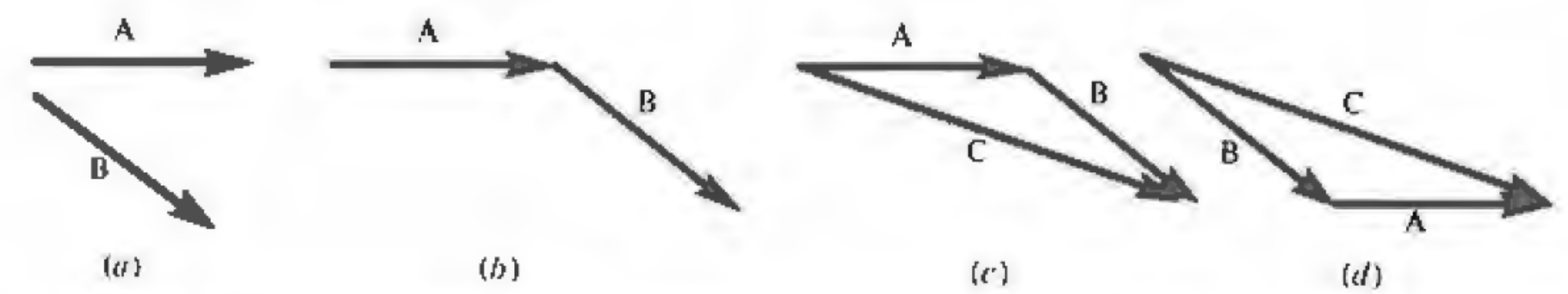


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

Addition of vectors

When adding vectors, their directions must be considered.

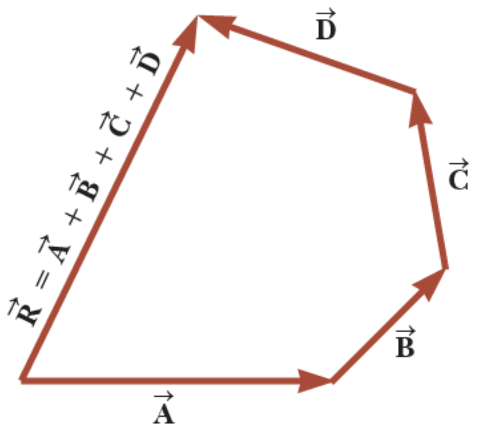
- first draw vector $\vec{\mathbf{A}}$ on graph paper, with its magnitude represented by a convenient length scale.
- then draw vector $\vec{\mathbf{B}}$ to the same scale, with its tail starting from the tip of $\vec{\mathbf{A}}$, as shown in the figure.
- The **resultant vector** $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the vector drawn from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

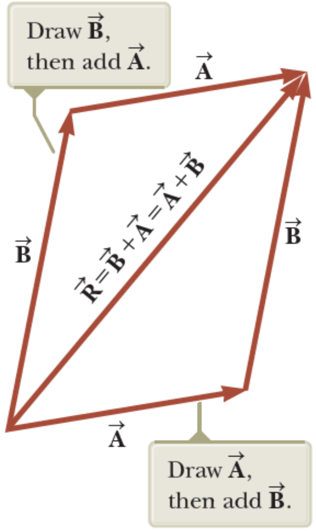


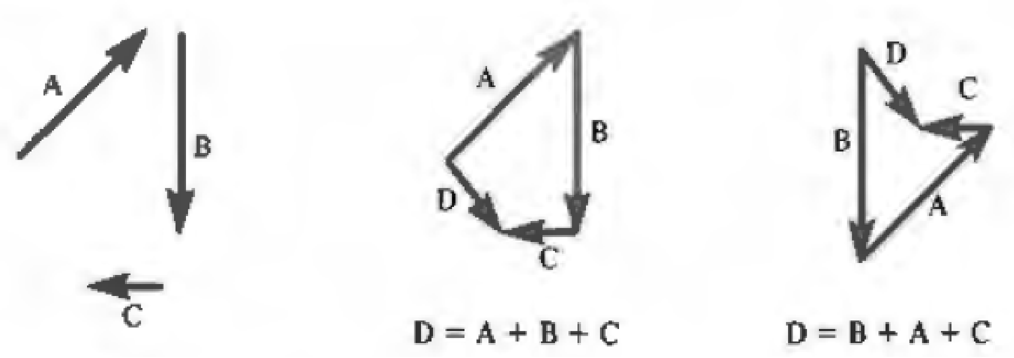
2.1 An introduction to vectors

- A geometric construction can also be used to add more than two vectors as shown in the Figure for the case of four vectors.
- The resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$ is the vector that completes the polygon.
- The resultant vector $\vec{\mathbf{R}}$ is the vector drawn from the tail of the first vector to the tip of the last vector.
- **Commutative law of addition:** sum of vectors is independent of the order of the addition.
- $$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$
- **Associative Property of Addition**:
  When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped.

$$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}$$





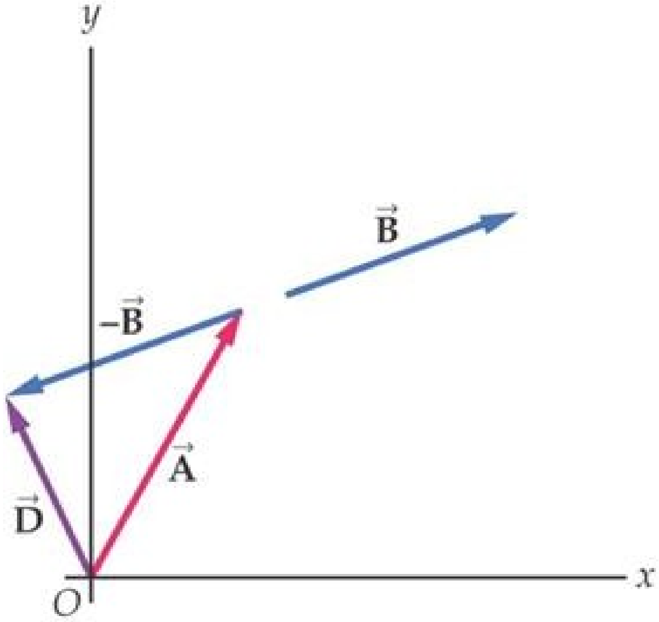


2.1 An introduction to vectors

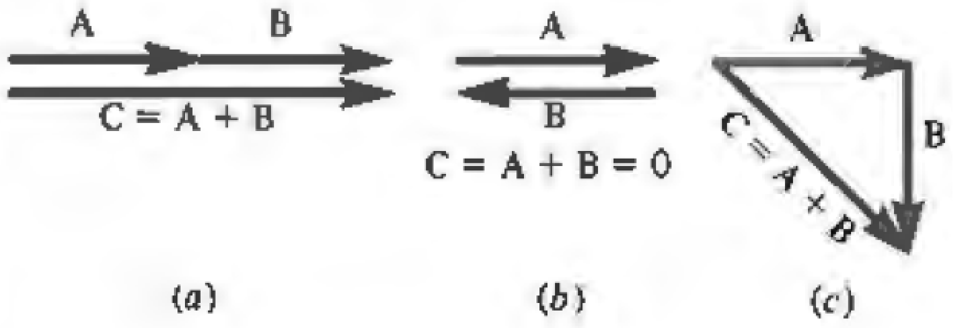
- **Negative of a vector**
  The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero. The vectors $\vec{\mathbf{A}}$ and $-\vec{\mathbf{A}}$ have the same magnitude but point in opposite directions.

- **Subtracting vectors:**
- Subtracting vectors is a special case of addition:
- $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is defined as the vector $-\vec{\mathbf{B}}$ added to vector $\vec{\mathbf{A}}$

$$\vec{D} = \vec{A} + (-\vec{B}) = \vec{\mathrm{A}} - \vec{\mathrm{B}}$$



- As example: the sum of two displacement vectors: (a) parallel, (b) equal in magnitude and opposite indirection and (c) perpendicular is shown in the figure



2.1 An introduction to vectors

Multiplying a Vector by a Scalar:

If vector $\vec{\mathbf{A}}$ is multiplied by a positive scalar quantity m, the product $m\vec{\mathbf{A}}$ is a vector that has the same direction as $\vec{\mathbf{A}}$ and magnitude mA.
If vector $\vec{\mathbf{A}}$ is multiplied by a negative scalar quantity $-m$, the product $-m\vec{\mathbf{A}}$ is directed opposite **A** .

Example:

The vector $5\vec{\mathbf{A}}$ is five times as long as $\vec{\mathbf{A}}$ and points in the same direction as $\vec{\mathbf{A}}$

The vector $-\frac{1}{3}\vec{\mathbf{A}}$ is one-third the length of $\vec{\mathbf{A}}$ and points in the direction opposite $\vec{\mathbf{A}}$.

2.1 An introduction to vectors

Vector components:

Definition of vector: A vector is defined with a **magnitude** and a **direction**. The magnitude is given by the length of the vector and the direction by a positive angle (Fig. 2.2 a)

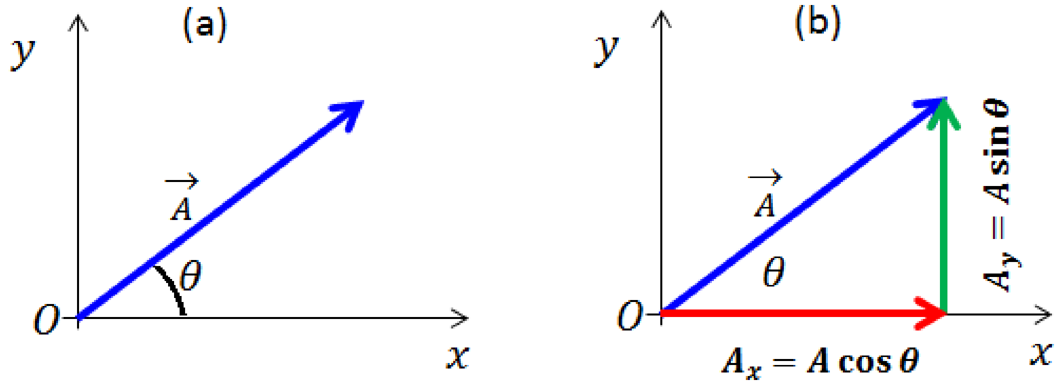


Figure 2.2: Components of vector

- A vector $\mathbf{A}$ is resolved in (x, y) plane into two components (Fig. 2.2 b) :
  - A_x is the component on the x axis
  - A_y is the component on the y axis, where:

$$A_x = A\cos\theta \quad and \quad A_y = A\sin\theta$$

2.1 An introduction to vectors

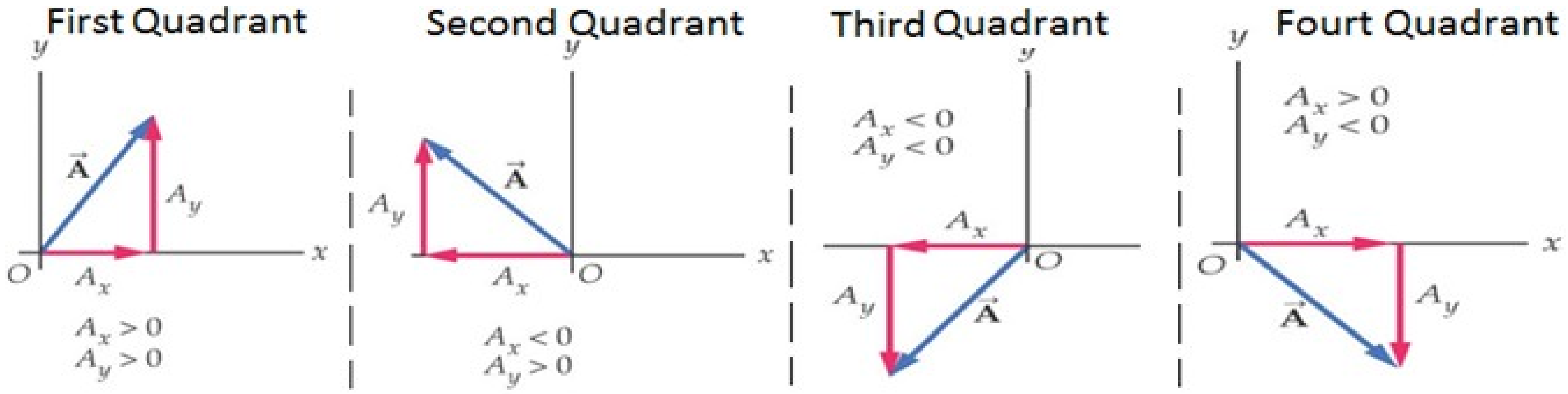
Vector components:

If the vector is defined in the (x, y) plane by its components $A_x$and A_y, then the magnitude and the direction of the vector are:

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) + C$$

C is a correction of the angle which depend on the quadrant where the vector is located

$C = 0$ if *the vector is in the second quadrant*
$C = 180°$ if *the vector is in the second quadrant*
$C = 180°$ if *the vector is in the third quadrant*
$C = 360°$ if *the vector is in the fourth quadrant*



2.1 An introduction to vectors

Vector components:

Example 2.1 page 30:

A person walks 1 km due east. If the person then walks a second kilometer, what is the final distance from the starting point if the second kilometer is walked :

(a) due east; (b) due west; (c) due south?

We will call the first displacement A and the second B .

Solution

We construct the sum $\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B}$ for the three cases (See Figure below).

(a) Since $\boldsymbol{A}$ and $\boldsymbol{B}$ are in the same direction,

$C = A + B = 2A = 2\ km.$

The vector $\boldsymbol{C}$ is directed due east.

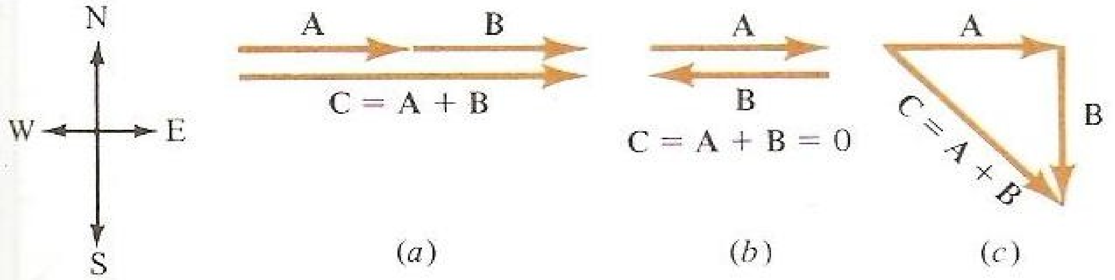


Figure 2.3. The sum of two displacement vectors that are equal in magnitude and (a) parallel; (b) opposite or antiparallel; and (c) perpendicular.

(b)Here, the vectors are opposite, so

$C = A – B = 0.$

(b) From the Pythagorean theorem:

$C^2 = A^2 + B^2 = 2\ A^2,$

so $C = \sqrt{2}$ A $= \sqrt{2}\ km$

$\boldsymbol{C}$ points toward the southeast (Fig.2.3)

2.1 An introduction to vectors

Vector components:

Example 2.2 page 32:

Find the components of the vectors **A** and $\boldsymbol{B}$ in Fig.2.7, if $A = 2$ and $B = 3$.

Solution:

A_x and A_y are positive:

$$A_x = A\cos\theta = 2\cos 30° = 2(0.866) = 1.73$$

$$A_y = A\sin\theta = 2\sin 30° = 2\,(0.500) = 1.00$$

$$\boldsymbol{A} = 1.73\hat{x} + 1.00\hat{\boldsymbol{y}}$$

From Fig.2.7b, B_x is positive and B_y is negative:

$\cos 45° = \sin 45° = 0.707$,

$B_x = 3\cos 45° = 3\,(0.707) = 2.12$

$B_y = -3\sin 45° = -3\,(0.707) = -2.12$

$\boldsymbol{B} = 2.12\hat{x} - 2.12\hat{y}$

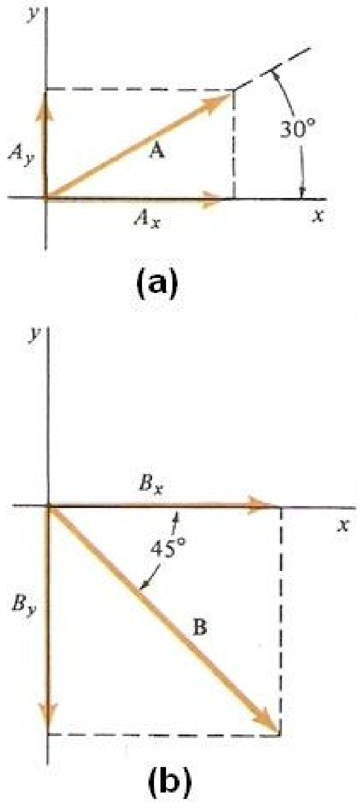


Figure 2.7

2.1 An introduction to vectors

Adding vectors using components:

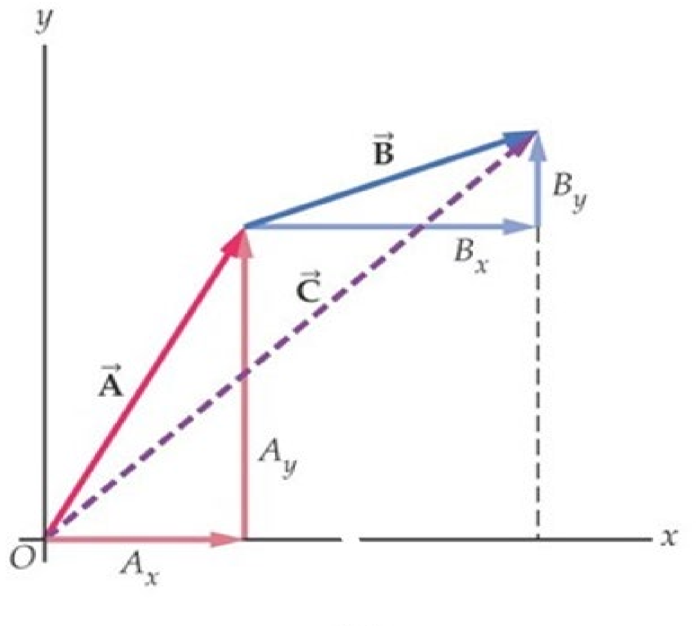
1. Find the components of each vector to be added
2. Add the $x-$ and $y-$components separately
3. Find the resultant vector.

$$\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$$

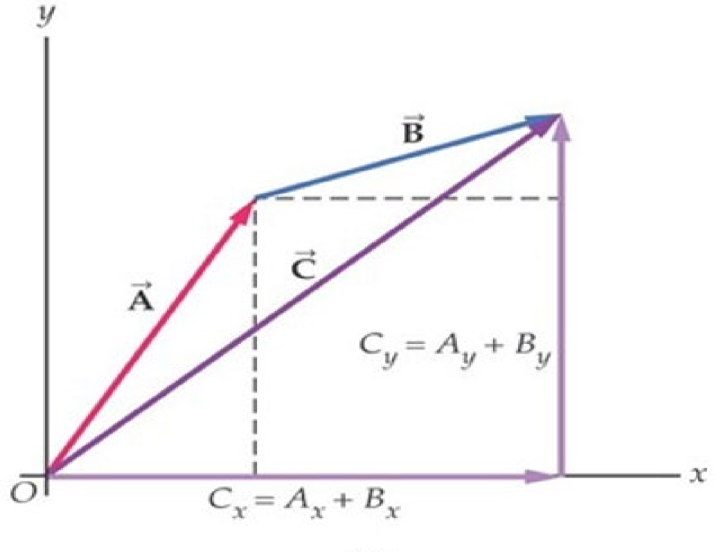
$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$

$$C = \sqrt{A_x^2 + B_y^2}$$



(a)



(b)

2.1 An introduction to vectors

Example 2.3 page 33:

$\vec{A} = 2\hat{x} + \hat{y}, \quad \vec{B} = 4\hat{x} + 7\hat{y}$

(a) Find the components of $\vec{C} = \vec{A} + \vec{B}$

(b) Find the magnitude of $\vec{C}$ and its angle θ with respect to the positive x axis

Solution:

a) Using the equation $\vec{C} = (A_x + B_x)\,\hat{x} + (A_y + B_y)\hat{y}$

We can write $\vec{C} = (2+4)\hat{x} + (1+7)\hat{y} = 6\hat{x} + 8\hat{y}$

Thus $C_x = 6$, and $C_y = 8$.

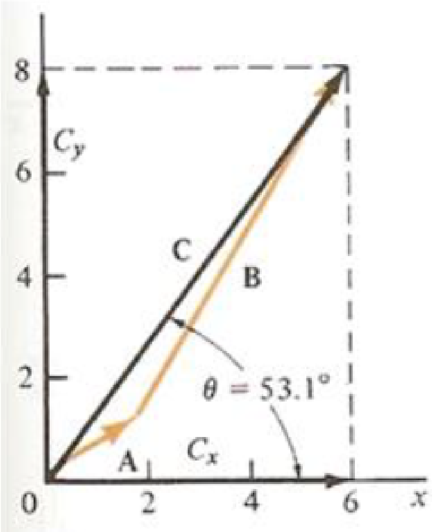


Figure 2.9.

(b) From the Pythagorean theorem: $C^2 = {C_x}^2 + {C_y}^2 = 6^2 + 8^2 = 100$

Then $C = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{100} = 10$

From Fig. 2.9, we see that the angle θ satisfies:

$$tan\theta = \frac{C_y}{C_x} = \frac{8}{6} = 1.33$$

Then $\theta = tan^{-1}(1.33) = 53.1°$

2.1 An introduction to vectors

Example:

Let $\boldsymbol{A} = 2\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}$ and $\mathbf{B} = 4\hat{\boldsymbol{x}} - 7\hat{\boldsymbol{y}}$, Find

(a) $\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B}$.

$$\boldsymbol{C} = (2\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}) + (4\hat{x} - 7\hat{\boldsymbol{y}}) = 6\hat{\boldsymbol{x}} - 6\hat{\boldsymbol{y}}$$

(b) $\boldsymbol{D} = 2\boldsymbol{A} - \frac{2}{3}\boldsymbol{B}$

$$\boldsymbol{D} = 2(2\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}) - \frac{2}{3}(4\hat{\boldsymbol{x}} - 7\hat{\boldsymbol{y}}) = -2\hat{\boldsymbol{x}} + 12.5\hat{\boldsymbol{y}}$$

(c) $|\boldsymbol{A}|$, $|\boldsymbol{B}|$, $|\boldsymbol{C}|$, and $|\boldsymbol{D}|$

$$|\boldsymbol{A}| = \sqrt{2^2 + 1^2} = 2.23$$

$$|\boldsymbol{B}| = \sqrt{4^2 + (-7)^2} = 8.06$$

$$|\boldsymbol{C}| = \sqrt{6^2 + (-6)^2} = 8.49$$

$$|\boldsymbol{D}| = \sqrt{(-2)^2 + (12.5)^2} = 12.65$$

(d) The angle that $\boldsymbol{C}$ makes with the positive x-axis.

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-6}{6}\right) + 180° = -45° + 180° = 135°$$, θ is in the second quadrant

(e) The angle that $\boldsymbol{B}$ makes with the positive x-axis

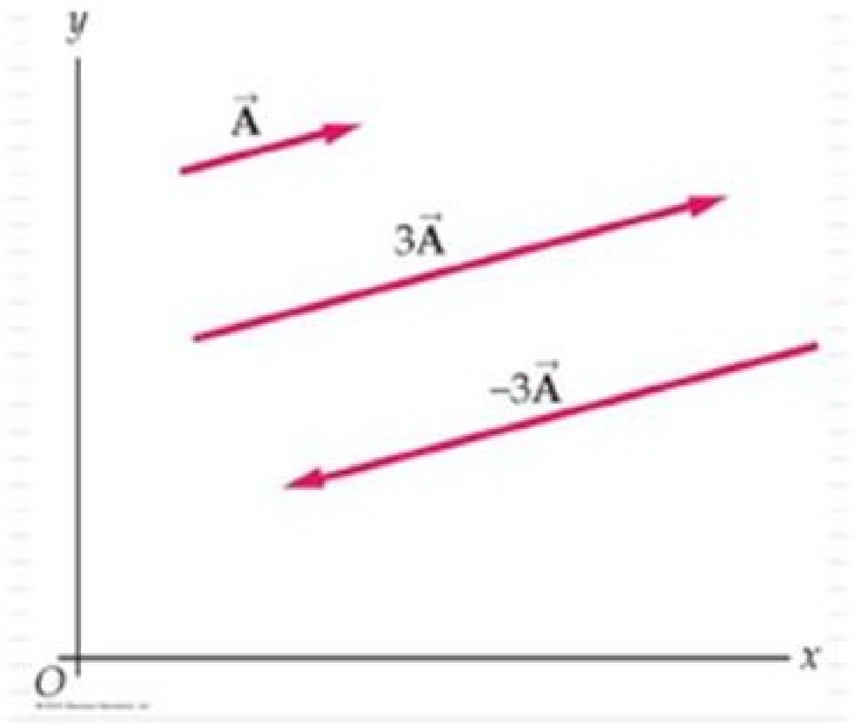
$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{-7}{4}\right) + 360° = 299.7°$$, θ is in the fourth quadrant

2.1 An introduction to vectors

Example

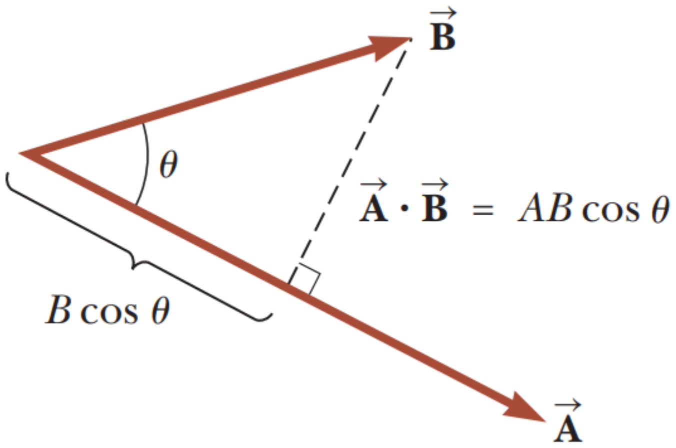
Multiplying a vector by 3 increases its magnitude by a factor of 3, but does not change its direction.

$$\vec{A} = A_x\hat{x} + A_y\hat{y}$$
$$3\vec{A} = 3A_x\hat{x} + 3A_y\hat{y}$$
$$-3\vec{A} = -3A_x\hat{x} - 3A_y\hat{y}$$



2.1 An introduction to vectors

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:



$$\vec{A} \cdot \vec{B} \equiv AB\cos\theta$$

The vector (cross) product of $\vec{A}$ and $\vec{B}$ is defined as a *third vector*, $\vec{C}$, where

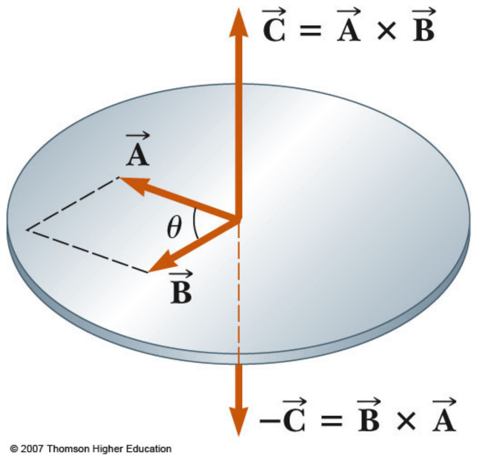
$$\vec{C} = \vec{A} \times \vec{B},$$

Direction of $\vec{C}$ is a vector perpendicular to both $\vec{A}$ and $\vec{B}$.

Direction of $\vec{C}$ determined by the right-hand rule.

Magnitude of $\vec{C}$ is

$$C = AB\sin\theta$$



Right-hand rule

2.1 An introduction to vectors

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\cos\theta$$

The vector (cross) product of $\vec{A}$ and $\vec{B}$ is defined as a *third vector*, $\vec{C}$, where

$$\vec{C} = \vec{A} \times \vec{B},$$

Direction of $\vec{C}$ is a vector perpendicular to both $\vec{A}$ and $\vec{B}$.

Direction of $\vec{C}$ determined by the right-hand rule.

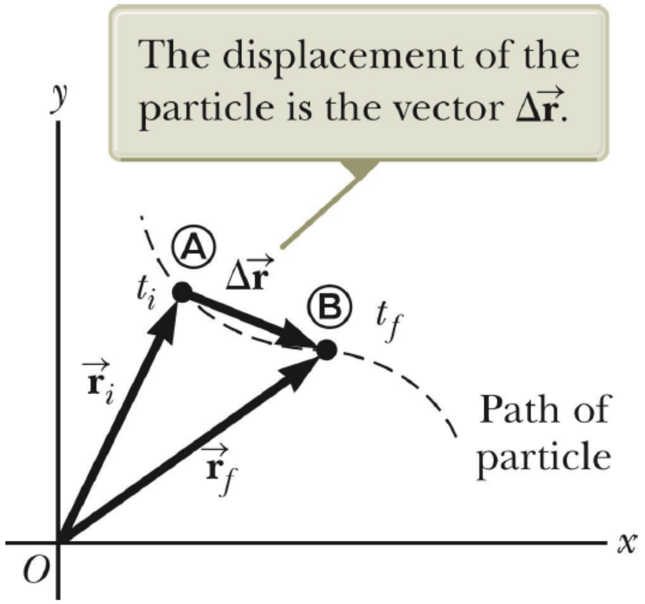
Magnitude of $\vec{C}$ is

$$C = ABsin\theta$$

2.2 The velocity in two dimensions

- In two dimensions, position, velocity, and acceleration are presented by vectors: motion in a plane.
- A problem involving motion in a plane is a pair of one-dimensional motion problems.
- The **position** of an object is described by its position vector, $\vec{\mathbf{r}}$.
  - $\vec{\mathbf{r}}$ is drawn from the origin and has two components:

$$\vec{\mathbf{r}} = x\hat{\boldsymbol{x}} + y\hat{y}$$



- The **displacement** of the object is defined as the *change in its position*:

$$\Delta\vec{\boldsymbol{s}} = \vec{\boldsymbol{s}}_f - \vec{\boldsymbol{s}}_i$$
$$= (x_f - x_i)\hat{\boldsymbol{x}} + (y_f - y_i)\hat{\boldsymbol{y}}$$
$$= \Delta x\,\hat{\boldsymbol{x}} + \Delta y\,\hat{\boldsymbol{y}}$$

where $\Delta x = x_f - x_i$ and $\Delta y = y_f - y_i$

2.2 The velocity in two dimensions

- The **average velocity** is the ratio of the displacement to the time interval for the displacement:
  - It is parallel to the displacement $\Delta\vec{s}$

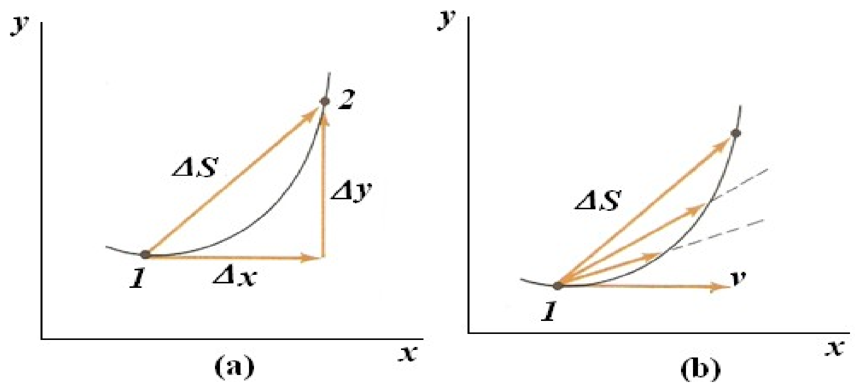


Figure 2.10 (a) An object moves along a path in a plane . At time t_1 is at point 1 and at t_2 is at point 2, the average velocity is parallel to $\Delta\vec{s}$. (b) As the time interval $t_2 - t_1$ becomes smaller , so does $\Delta\vec{s}$, the average velocity approaches the instantaneous velocity v at t which is tangent to the path at point 1.

If the displacement in a time interval Δt is denoted by the vector $\overrightarrow{\Delta s}$, ***then the average velocity** of the*

object is parallel to $\overrightarrow{\Delta s}$ and is given by: $\vec{\bar{v}} = \dfrac{\Delta\vec{s}}{\Delta t}$

where $\Delta\vec{s} = \Delta x\,\hat{x} + \Delta y\,\hat{y}$, ther $\vec{\bar{v}} = \dfrac{\Delta x}{\Delta t}\hat{x} + \dfrac{\Delta y}{\Delta t}\hat{y}$

Thus, the components of $\vec{\bar{v}}$ are

$$\bar{v}_x = \frac{\Delta x}{\Delta t}, \qquad \bar{v}_y = \frac{\Delta y}{\Delta t}$$

2.2 The velocity in two dimensions

The **instantaneous velocity** is the is the average velocity for an extremely short time interval

$$\vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{s}}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{x}} + \frac{dy}{dt}\hat{\boldsymbol{y}} = v_x\hat{\boldsymbol{x}} + v_y\hat{\boldsymbol{y}}$$

Thus, the components of $\vec{\boldsymbol{v}}$ are

$$v_x = \frac{dx}{dt}, \qquad v_y = \frac{dy}{dt}$$

2.2 The velocity in two dimensions

Example :

An ambulance travels from the hospital $10\ km$ due south in $7\ min$, and then $5\ km$ due east in $3\ min$. Find (a) the final position of the ambulance, (b) its average velocity

Solution

(a) $\overrightarrow{\Delta S} = 5\ km\ \hat{x}\ - 10\ km\ \ \hat{y}$

(b) $\Delta t\ = 7min + 3min = 10\ min = 0.167\ hour$

then:

$$\vec{v} = \frac{\overrightarrow{\Delta S}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{x} + \frac{\Delta y}{\Delta t}\hat{y} = \frac{5\ km}{0.167h}\ \hat{x} - \frac{10\ km}{0.167h}\ \hat{y} = 30\ \ km/h\ \ \hat{x} - 60\ \ km/h\ \ \hat{y}$$

2.2 The velocity in two dimensions

Example 2.4 page 34:

A car travels halfway around an oval racetrack at a constant speed of $30\ ms^{-1}$ (Fig. 2.11).

(a) What are its instantaneous velocities at points 1 and 2?

(b) It takes 40 s to go from 1 to 2, and these points are $300\ m$ apart. What is the average velocity of the car during this time interval?

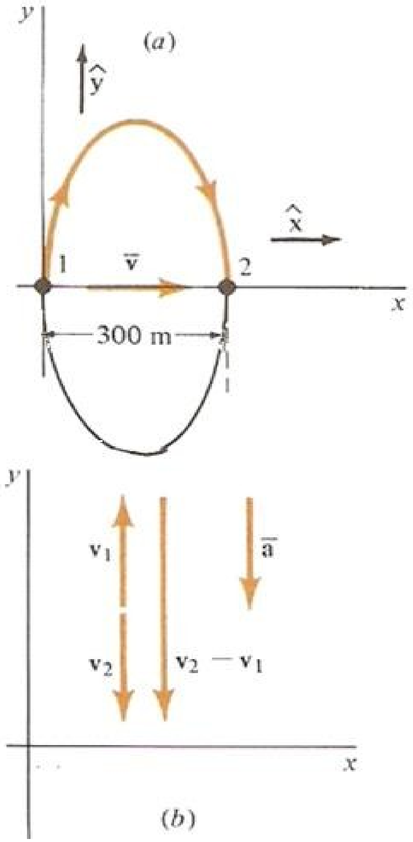
Solution:

a) The instantaneous velocity is tangent to the path of the car, and its magnitude is equal to the speed. Thus, at point 1 the velocity is directed in the $+y$ direction and $\overrightarrow{v_1} = 30\ ms^{-1}\hat{y}$. Similarly, at point 2 the velocity is long the $-y$ direction, and $\overrightarrow{v_2} = -30\ ms^{-1}\hat{y}$

(b) The average velocity is the displacement divided by the elapsed time. The displacement is entirely along the x direction, so $\Delta\vec{S} = 300\ m\ \hat{x}$.

Since Δt = 40 s, $\bar{\vec{v}} = \frac{\Delta\vec{S}}{\Delta t} = \frac{300\ m}{40\ s}\hat{x} = 7.5\ ms^{-1}\ \hat{x}$

The average velocity during this time interval is directed along the $+x$ axis. Its magnitude is less than the speed of $30\ ms^{-1}$ because the car does not travel in a straight line.



2.2 The acceleration in two dimensions

The **average acceleration** is defined by :

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

The **instantaneous acceleration** a is the average acceleration during an extremely short interval

$$\vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt} = \frac{dv_x}{dt}\hat{\boldsymbol{x}} + \frac{dv_y}{dt}\hat{\boldsymbol{y}} = a_x\hat{\boldsymbol{x}} + a_y\hat{\boldsymbol{y}}$$

Thus, the components of $\vec{\boldsymbol{v}}$ are

$$a_x = \frac{dv_x}{dt} \text{ , } a_y = \frac{dv_y}{dt}$$

2.2 The acceleration in two dimensions

Example 2.6 page 35:

In Exercise 2.4 the velocity of the car changed from $\vec{v_1} = 30\ m\ s^{-1}\ \hat{y}$ to $\vec{v_2} = -30\ m\ s^{-1}\ \hat{y}$ in 40 s.

What was the average acceleration of the car in that time interval?

Solution :

The average acceleration is defined as the velocity change divided by elapsed time:

$$\vec{a} = \frac{\vec{v_2} - \vec{v_1}}{\Delta t} = \frac{-30\ m\ s^{-1}\ \hat{y} - 30\ m s^{-1}\ \hat{y}}{40} = -1.5\ m\ s^{-2}\ \hat{y}$$

Thus the average acceleration during the time the car goes from point 1 to point 2 is directed in the $-y$ direction, or downward in Fig. (2.11b)

2.2 The acceleration in two dimensions

The previous exercise illustrates two important points:

1 If the velocity is constant, the acceleration is zero , since $\boldsymbol{a}$ is the rate of change of the velocity.

However, when the speed is constant, the acceleration may or may not be zero. If an object moves at a constant speed along a curved path, its velocity is changing direction, and it is accelerating. We feel the effects of this acceleration when a car turns quickly.

The acceleration is zero only when the speed and direction of motion are both constants.

2 The directions of the velocity and acceleration at any instant can be related in many ways.

The magnitude and direction of $\boldsymbol{a}$ are determined by how $\boldsymbol{v}$ is changing.

When a car moves along a straight road, the acceleration is parallel to the velocity if v is increasing and opposite if v is decreasing.

When the motion is along a curved path, the acceleration is at some angle to the velocity.

2.2 The acceleration in two dimensions

Example: the position of an object that is moving in the x-y plane varies with time as:

$x(t) = 2t^2 + 1, \;\; y(t) = t^3$

where x and y are in meters ant t is in seconds. Find:

(1) the position at $t = 1$ s and $t = 2$ s.

(2)The average velocity in the time interval $[1,2]$ s.

(3) The instantaneous velocity at $t = 1$ s and $t = 3$ s.

(4) The average acceleration in the time interval $[1,3]$ s

(5) The instantaneous acceleration at $t = 3$ s.

Answer:

(1) $x(t = 1\ s) = 2(1)^2 + 1 = 3\ m$

$y(t = 1\ s) = (1)^3 = 1\ m$

$\mathbf{s}(\mathrm{t} = 1\ \mathrm{s}) = (3\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})\ m$

$x(t = 2\ s) = 2(2)^2 + 1 = 9\ m$

$y(t = 2\ s) = (2)^3 = 8\ m$

$\mathbf{s}(\mathrm{t} = 2\ \mathrm{s}) = (9\hat{\boldsymbol{x}} + 8\hat{\boldsymbol{y}})\ m$

(2) $\bar{\boldsymbol{v}} = \frac{\Delta x}{\Delta t}\hat{\boldsymbol{x}} + \frac{\Delta y}{\Delta t}\hat{\boldsymbol{y}} = \frac{(9-3)m}{(2-1)\ s}\hat{\boldsymbol{x}} + \frac{(8-1)m}{(2-1)\ s}\hat{\boldsymbol{y}}$

$\bar{\boldsymbol{v}} = (6\hat{\boldsymbol{x}} + 7\hat{\boldsymbol{y}})m/s$

(3) The instantaneous velocity is

$\vec{\boldsymbol{v}} = \frac{d\vec{s}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{x}} + \frac{dy}{dt}\hat{\boldsymbol{y}} = v_x\hat{\boldsymbol{x}} + v_y\hat{\boldsymbol{y}}$

$v_x = \frac{dx}{dt} = 4t$,

$v_x(t = 1\ s) = 4\ m/s, \qquad v_x(t = 3\ s) = 12\ m/s$

$v_y = \frac{dy}{dt} = 3t^2,$

$v_y(t = 1\ s) = 3\ m/s\ , \qquad v_y(t = 3\ s) = 27\ m/s$

(4) $\bar{a}_x = \frac{v_x(t=3\ s) - v_x(t=1\ s)}{(3-1)s} = \frac{(12-4)m/s}{2\ s} = 4\ m/s^2$

$\bar{a}_y = \frac{v_y(t=3\ s) - v_y(t=1\ s)}{(3-1)s} = \frac{(27-3)m/s}{2\ s} = 12\ m/s^2$

$\bar{\boldsymbol{a}} = (4\hat{\boldsymbol{x}} + 12\hat{\boldsymbol{y}})\ m/s^2$

(5) $a_x = \frac{dv_x}{dt} = 4\frac{m}{s^2} \qquad a_x(t = 3\ s) = 4\ m/s^2$

$a_y = \frac{dv_y}{dt} = 6t \qquad a_y(t = 3\ s) = 18\ m/s^2$

$\boldsymbol{a}(t = 3\ s) = (4\hat{\boldsymbol{x}} + 18\hat{\boldsymbol{y}})\ m/s^2$

2.4 Finding the motion of an object

- When the acceleration is constant, we can find the equations of motion. In this case the average and the instantaneous accelerations are equal.
- When a two-dimensional motion has a constant acceleration then the x- and y-components of the acceleration are constants:

  $\vec{\boldsymbol{a}} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} = \text{constant} \quad \Rightarrow \quad a_x = \text{constant and } a_y = \text{constant}$

- The following equations of motion with constant acceleration are obtained:

Relation number	x- component	y-compnent
1	$v_{x_f} = v_{x_i} + a_x\Delta t$	$v_{y_f} = v_{y_i} + a_y(\Delta t)$
3	$x_f - x_i = v_{x_i}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2$	$y_f - y_i = v_{y_i}(\Delta t) + \frac{1}{2}a_y(\Delta t)^2$
4	$v_{x_f}^2 = v_{x_i}^2 + 2a_x(x_f - x_i)$	$v_y^2 = v_{y_i}^2 + 2a_y(y_f - y_i)$

2.2 The acceleration in two dimensions

Example: A particle starts from the origin at $t = 0$ with initial velocity $\boldsymbol{v} = (20\hat{\boldsymbol{x}} - 15\hat{\boldsymbol{y}})\ m/s$. The particle moves in the x-y plane with an acceleration of a $= (4\hat{\boldsymbol{x}})\ m/s^2$.

(a) Determine the velocity vector at any time.

(b) Calculate the velocity and speed of the particle at $t = 5$ seconds.

(c) Determine the x and y coordinates of the particle at $t = 5$ seconds.

Answer: We are given that:

$x_i = 0, \qquad y_i = 0$

$v_{x_i} = 20\ m/s\ , \quad v_{y_i} = -15\ m/s$

$a_x = 4\ m/s^2, \quad a_y = 0$

(a) $v_{x_f} = v_{x_i} + a_x \Delta t = 20 + 4\text{t}$

$v_{y_f} = v_{y_i} + a_y \Delta t = -15 + 0 = -15\ m/s^2$

$\boldsymbol{v} = v_x \hat{\boldsymbol{x}} + v_y \hat{\boldsymbol{y}} = (20 + 4t)\hat{\boldsymbol{x}} - 15\hat{\boldsymbol{y}}$

(b) $\boldsymbol{v}(t = 5\ s) = (20 + 4 \times 5)\hat{\boldsymbol{x}} - 15\hat{\boldsymbol{y}} = (40\hat{\boldsymbol{x}} - 15\hat{\boldsymbol{y}})\ m/s^2$

$v = \sqrt{40^2 + (-15)^2}\ m/s = 43\ m/s$

(c) $x_f - x_i = v_{x_i}(\Delta t) + \frac{1}{2} a_x (\Delta t)^2$

$x_f - 0 = 20 \times 5 + \frac{1}{2} \times 4 \times 5^2$

$x_f = 150\ m$

$y_f - y_i = v_{y_i}(\Delta t) + \frac{1}{2} a_y (\Delta t)^2$

$y_f - 0 = -15 \times 5 + \frac{1}{2} \times 0 \times 5^2$

$y_f = -75\ m$

$\boldsymbol{r}_f = x_f \hat{\boldsymbol{x}} + y_f \hat{\boldsymbol{y}} = (150\hat{\boldsymbol{x}} - 75\hat{\boldsymbol{y}})\ m$

Chapter 2 : Motion in two dimensions

① Vector $\vec{A}$ has magnetude of 8 m and makes 45° with positive x-axes, and $\vec{B}$ with magnetude 8 and directed along the negative y-axes.

(a) use graphical method to find $\vec{A}+\vec{B}$

$\vec{A}-\vec{B}$

(b) resolve the components and find $\vec{A}+\vec{B}$; $\vec{A}-\vec{B}$

② consider $\vec{A} = 3\hat{x} - 2\hat{y}$; $\vec{B} = -\hat{x} - 4\hat{y}$

find (a) $\vec{A}+\vec{B}$, $\vec{A}-2\vec{B}$;

(b) $|\vec{A}+\vec{B}|$; $|\vec{A}-2\vec{B}|$

(d) the direction of $(\vec{A}+\vec{B})$ and $(\vec{A}-2\vec{B})$

(3) the position vector $\vec{S}$ of a particle varies with time according to the equation $\vec{S} = 3\hat{x} - 6t^2\hat{y}$

find $\vec{v}$, $\vec{a}$ and determine the position and velocity and acceleration at $t = 1$ sec. (magnitude and direction)

(4) a mass initially located at the origin has an acceleration of $\vec{a} = 3\hat{y}$ m/s^2 and initial velocity $\vec{v} = 5\hat{x}$ (m/s)
find: the vector position and velocity at any time t and at $t = 1$ seconds