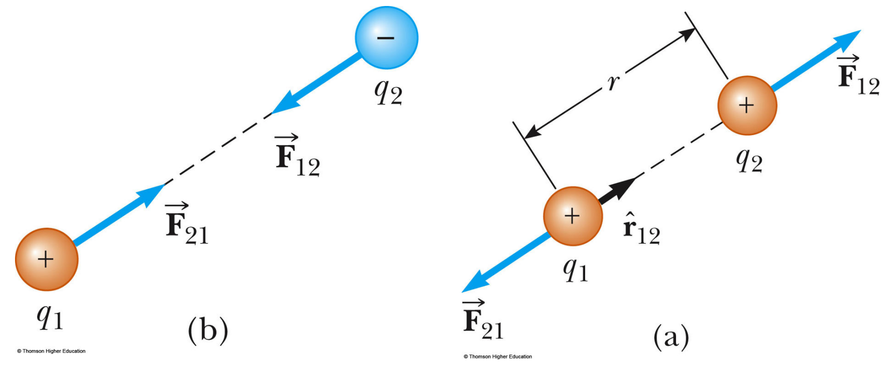
Chapter 16: Electric forces, fields and potentials
Chapter 17: Direct currents
Chapter 19: Magnetism

Dr. Ghassan Alna'washi

COURSE TOPICS:

16.1 Electric forces
16.2 Electric field
16.4 Electric potential
16.8 Capacitance
17.1 Electric current
17.2 Resistance
17.4 Power in electrical circuits
17.8 Electrical safety
19.1 Magnetic field
19.2 The magnetic force on a moving charge

16.1: Electric forces



- The Electric forces are produced because of the existence of charges.
- There are two kinds of charges: positive and negative.
- Like charges repel each other and unlike charges attract each other

- **Coulomb's law** states that the force between two electric charges (q and Q) is proportional to the product of the charges (qQ) and inversely proportional to their separation squared (r^2).

$$\boxed{\mathbf{F} = \frac{kqQ}{r^2}\,\hat{\mathbf{r}}}$$

- Here, $\hat{\mathbf{r}}$ is a unit vector directed toward q.
- In S.I. units, the charge is measured in Coulombs (C).
- k has the experimentally determined value

$$k = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

16.1: Electric forces

Example 16.1:

A positive charge q is near a positive charge Q and a negative charge $-Q$ (Fig. 16.2). (a) Find the magnitude : and direction of the force on q.(b) If $q = 10^{-6}\ C$, $Q = 2 \times 10^{-6}\ C$, and $a = 1\ m$, find the force on q.

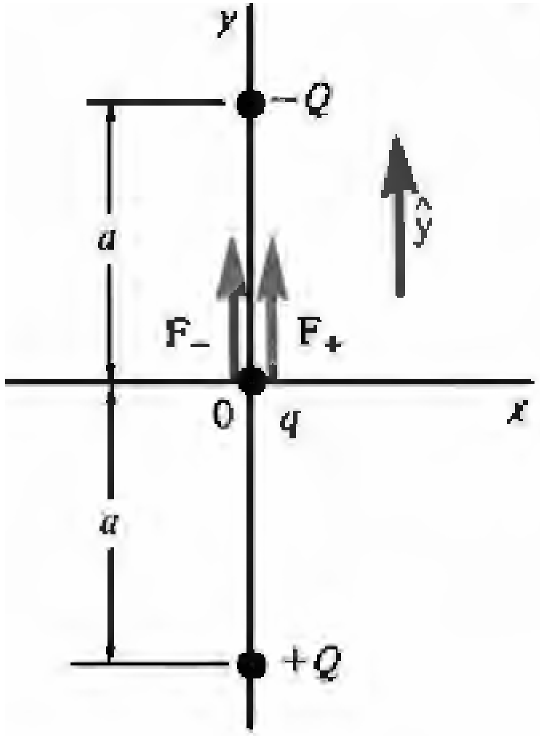
Answer:

(a) Since q is positive, it is repelled by the positive charge Q. Thus, the force $\boldsymbol{F}_+$ on q due to Q is directed away from Q, or in the $\hat{\mathbf{y}}$ direction. Similarly, q is attracted toward $-Q$, so the force $\boldsymbol{F}_-$ on q due to $-Q$ is directed toward from $-Q$, or in the $\hat{\mathbf{y}}$ direction. The directions are shown in the figure.

Thus, the force on q due to the charge Q is

$$\mathbf{F}_+ = \frac{kqQ}{a^2}\,\hat{\mathbf{y}}$$

Since both q and Q are positive, the force is parallel to $\hat{\mathbf{y}}$, or upward.



The force on q due to the charge Q is

$$\mathbf{F}_- = \frac{kq(-Q)}{a^2}(-\hat{\mathbf{y}}) = \frac{kqQ}{a^2}\,\hat{\mathbf{y}}$$

Since $\boldsymbol{F}_+$ and $\boldsymbol{F}_-$ are parallel, the net force on q is

$$\mathbf{F} = \mathbf{F}_+ + \mathbf{F}_- = \frac{2kqQ}{a^2}\,\hat{\mathbf{y}}$$

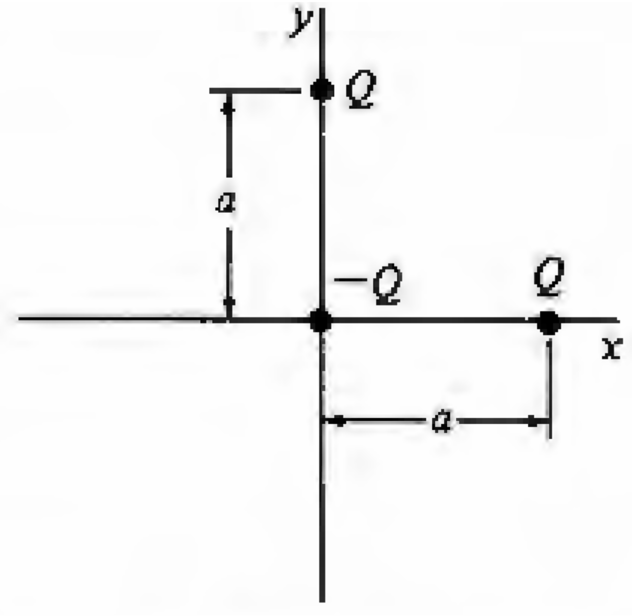
(b) Substituting the numerical values given, the net force on q is

$$\mathbf{F} = \frac{2kqQ}{a^2}\,\hat{\mathbf{y}} = \frac{2(9\times 10^9\ \mathrm{N\ m^2\ C^{-2}})(10^{-6}\ \mathrm{C})(2\times 10^{-6}\ \mathrm{C})}{(1\ \mathrm{m})^2}\,\hat{\mathbf{y}} = 3.6\times 10^{-2}\,\hat{\mathbf{y}}\ \mathrm{N}$$

16.1: Electric forces

Example

Find the magnitude and direction of the force on the charge – Q in the Figure.



Answer:

The charge $-Q$ is attracted toward both charges Q. Thus, the forces are

$$\boldsymbol{F}_{-Q} = \frac{k(-Q)Q}{a^2}(-\hat{\mathbf{x}}) + \frac{k(-Q)Q}{a^2}(-\hat{\mathbf{y}}) = \frac{kQ^2}{a^2}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

The magnitude and direction of the force are

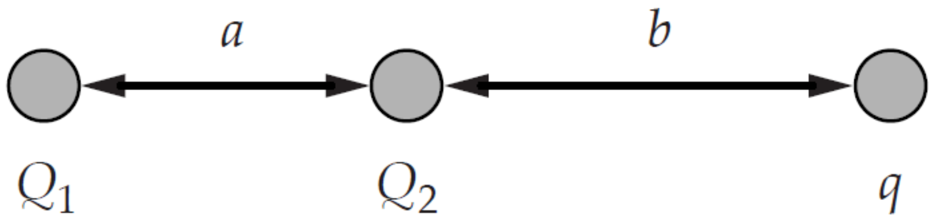
$$F = \frac{kQ^2}{a^2}\sqrt{1^2 + 1^2} = \frac{\sqrt{2}kQ^2}{a^2}$$

$$\theta = \tan^{-1}\left(\frac{kQ/a^2}{kQ/a^2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

16.1: Electric forces

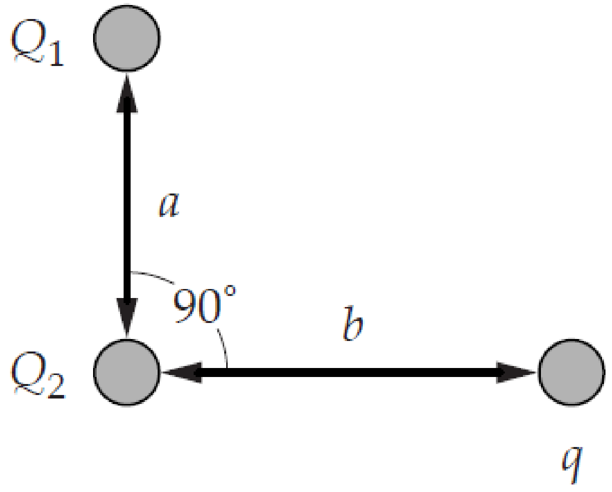
Problem:

If $a = 3.0$ mm, $b = 4.0$ mm, $Q_1 = -60\ nC$, $Q_2 = 80\ nC$, and $q = 30$ nC in the figure, what is the magnitude of the total electric force on q?



Problem:

If $a = 3.0$ mm, $b = 4.0$ mm, $Q_1 = 60\ nC$, $Q_2 = 80\ nC$, and $q = 24$ nC in the figure, what is the magnitude of the total electric force on Q_2?



16.2: Electric Field

- Electric field exist around charged objects.
- When we have one or more electric charges, we may say that they produce an electric field in their vicinity.
- If another charge q is present, it experiences a force proportional to the electric field $\boldsymbol{E}$ and to q itself:

$$\boxed{\mathbf{F} = q\mathbf{E}}$$

- If q is a positive charge, the force due to the electric field is parallel to the flield (Positive charges experience forces parallel to the field)
- . If q is negative, $\boldsymbol{F}$ is proportional to $-\boldsymbol{E}$, so the force is opposite to $\boldsymbol{E}$ (negative charges experience forces opposite to the field).
- The units of the electric field are those of force divided by charge. Thus, in S.I. units, the electric field has units of newtons per coulomb (N/C).
- The expression for the field due to a single point charge can be deduced from Coulomb's law. As we saw, the force on a charge q due to a charge Q at a distance r is

$$\mathbf{F} = \frac{kqQ}{r^2}\,\hat{\mathbf{r}}$$

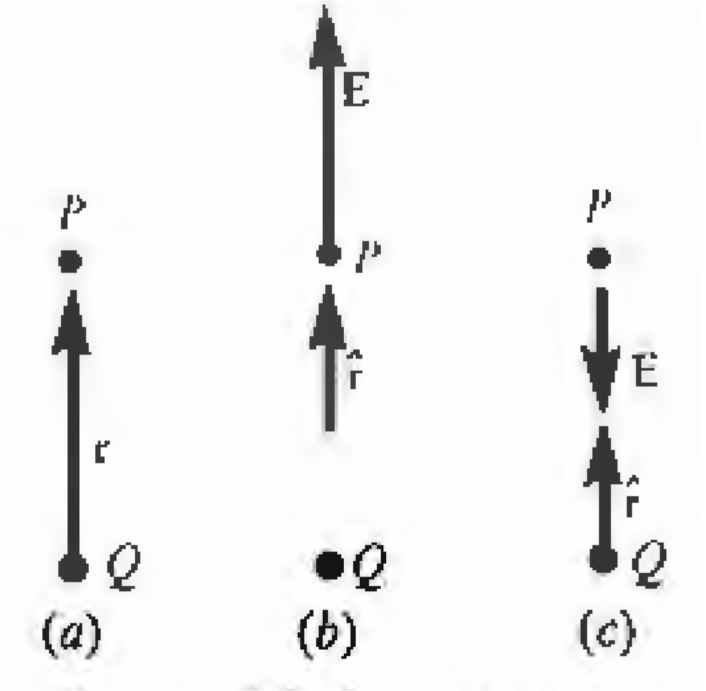
- Thus, it follows that at point P the field due to Q is $\boldsymbol{E} = \frac{\boldsymbol{F}}{q}$ or

$$\boxed{\mathbf{E} = \frac{kQ}{r^2}\,\hat{\mathbf{r}}}$$

field due to a single point charge

16.2: Electric Field

- If Q is positive, $\boldsymbol{E}$ points along $\hat{\mathbf{r}}$ or away from Q;
- If Q is negative, $\boldsymbol{E}$ points along $-\hat{\mathbf{r}}$ or toward Q.
  - The electric field due to a charge points away from the charge if it is positive, and toward it if the charge is negative.



- When there are several charges $Q_1, Q_2, \ldots\ldots$, at various positions, the electric field $\boldsymbol{E}$ at a point P is the vector sum of the individual electric fields $\boldsymbol{E}_1, \boldsymbol{E}_2, \ldots.$, due to all the charges:

$$\boxed{\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \cdots}$$

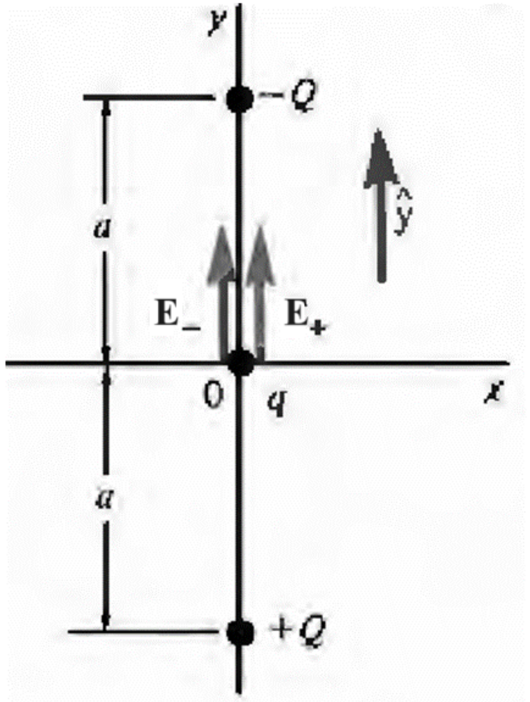
- If a charge q is placed at P, then the force on it is again given by $\boldsymbol{F} = q\boldsymbol{E}$.

Example,
a Na^+ ion has a charge q $= e = 1.6\ x\ 10^{-19}\ C$. If the field in a cell membrane is $10^6\ N/C$, Find the magnitude of the force on the ion.
Answer:

$$F = qE = (1.6 \times 10^{-19}\,\mathrm{C})(10^6\,\mathrm{N\,C^{-1}}) = 1.6 \times 10^{-13}\,\mathrm{N}$$

The force is directed along the electric field

16.2: Electric Field



Example 16.2

The charges Q and $-Q$ of example 16.1 are shown again in Fig. 16.6. (a) If $\mathrm{Q} = 2 \times 10^{-6}\ C$ and $a = 1\ m$, find the electric field at the origin. (b) Find the force on the charge $q = 10^{-6}\ C$ placed at the origin.

Answer:

(a) The field at the origin due the charge Q points away from the positive (upward)charge and is given by

$$\mathbf{E}_+ = \frac{kQ}{a^2}\,\hat{\mathbf{y}}$$

The field at the origin due the charge $-Q$ points toward the negative (upward) charge and is given by

$$\mathbf{E}_- = \frac{k(-Q)}{a^2}(-\hat{\mathbf{y}}) = \frac{kQ}{a^2}\,\hat{\mathbf{y}}$$

The net electric field at the origin is the sum,

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{2kQ}{a^2}\,\hat{\mathbf{y}} = \frac{2(9\times10^9\ \mathrm{N\ m^2\ C^{-2}})(2\times10^{-6}\ \mathrm{C})}{(1\ \mathrm{m})^2}\,\hat{\mathbf{y}} = 3.6\times10^4\ \hat{\mathbf{y}}\ \mathrm{N\ C^{-1}}$$

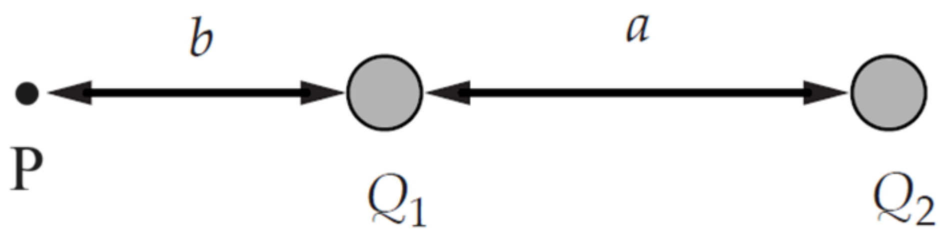
(b) The force on a charge $q = 10^{-6}\ C$ at the origin is

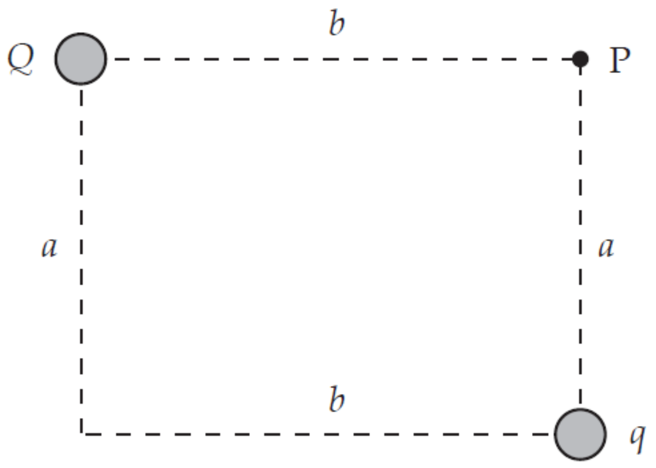
$$\mathbf{F} = q\mathbf{E} = (10^{-6}\ \mathrm{C})(3.6\times10^4)\ \hat{\mathbf{y}}\ \mathrm{N\ C^{-1}}$$
$$= 3.6\times10^{-2}\ \hat{\mathbf{y}}\ \mathrm{N}$$

16.1: Electric forces

Problem:

If $a = 1.0\ m$, $b = 0.5\ m$, $Q_1 = 1.0\ nC$, and $Q_2 = -5\ nC$,
(a) what is the magnitude of the electric field at point P?
(b) What is the electric field at the midpoint between the two charges



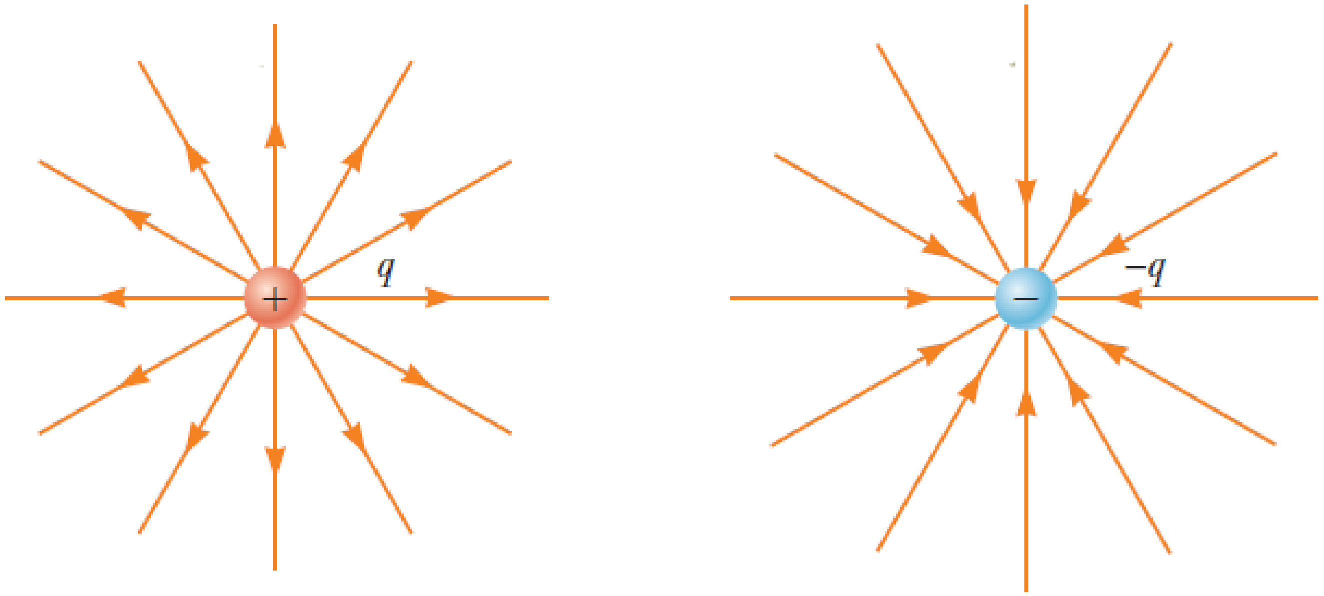


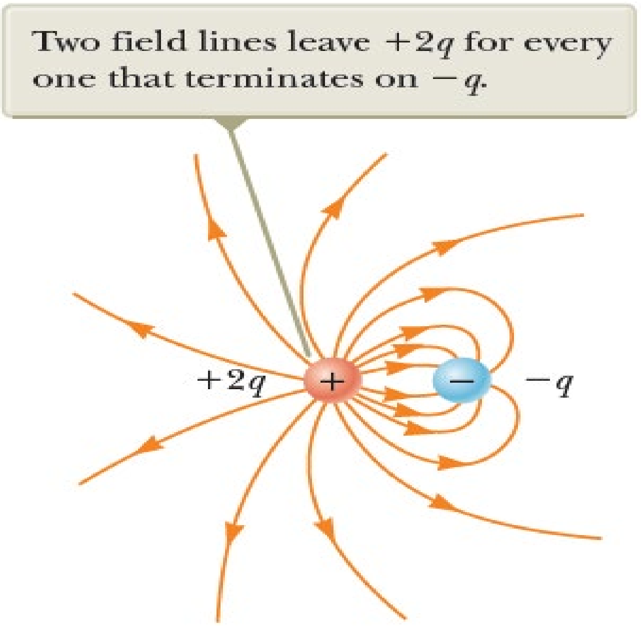
Problem:

If $a = 60\ cm$, $b = 80\ cm$, $Q = -4.0\ nC$, and $q = 1.5\ nC$, what is the magnitude of the electric field at point P?

Electric field diagram

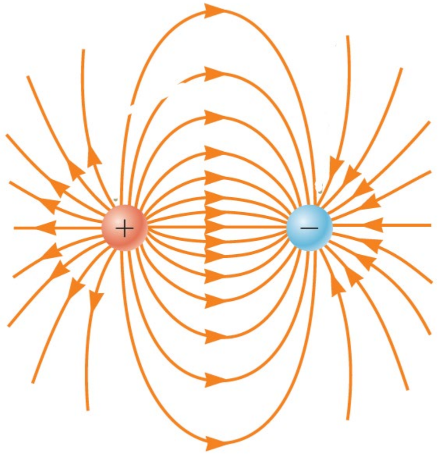
- Electric field line are graphical representation of the electric field
- The electric field lines have the following properties:
  1. The field lines points away from the positive charge and to ward the negative charge. In both cases the field becomes weaker as the distance from the charge increases, since it varies as $1/r^2$.
     At any point on an electric field line, its direction indicates the field direction at that point.
     The lines never cross, since there is a unique field direction everywhere.
  2. The closer the lines, the larger the electric field. The field lines are very close to each other near the charges where the field is large, and they spread out as the field becomes weaker farther from the charges.
  3. Electric field lines start at positive charges and end at negative charges. They never start or stop except at charges.
  4. Conventionally, the number of lines drawn is proportional to the size of the charges producing the field.



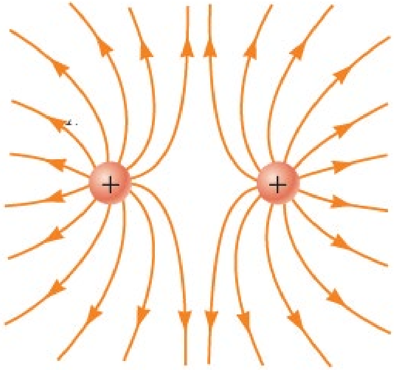


Electric field diagram

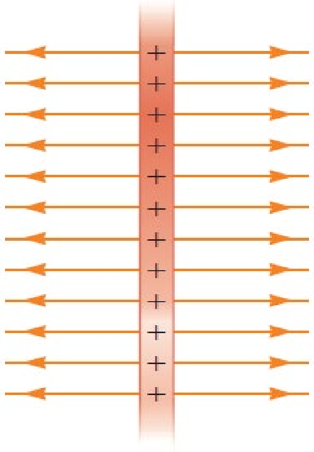
Below are diagrams of electric field lines of some charge distributions



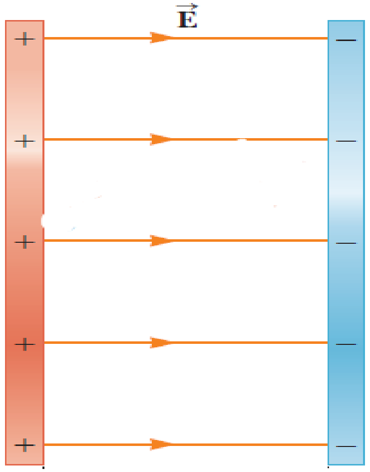
Two equal and opposite charges



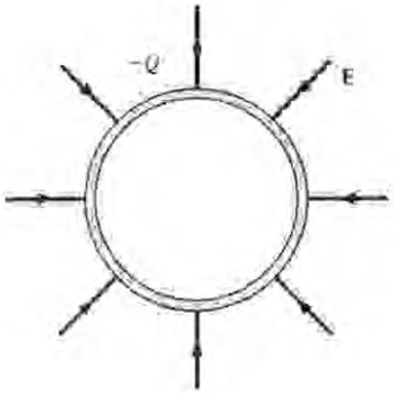
Two equal and positive charges



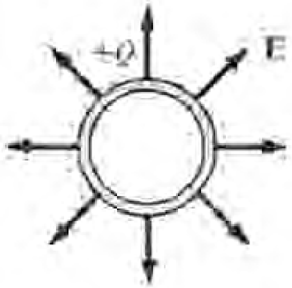
Positive infinite plate



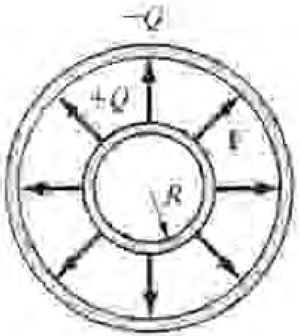
Infinite parallel plates of equal and opposite charges



Uniformly negatively charged spherical shell



Uniformly positively charged spherical shell



Two concentric charged spherical shell of equal and opposite charges

16.4 The electric potential

- The electric forces are conservative, so that their effects can be included in the potential energy of a system
- We introduce the **electrical potential** which **is the potential energy per unit charge**.
  - Suppose that at a certain position a charge q has an electrical potential energy $\mathcal{U}$. Then the electric potential V at that position is defined to be the potential energy divided by the charge,

$$\boxed{V = \frac{\mathcal{U}}{q}}$$

Type equation here.

  - The SI unit of potential is the volt (V), where from this definition 1 volt : 1 joule per coulomb.
- **The potential differences, ΔV,** are often referred to as voltages.

$$\boxed{\Delta V = \frac{\Delta \mathcal{U}}{q}}$$

- Thus, the change in the potential energy is

$$\boxed{\Delta \mathcal{U} = \mathcal{U} - \mathcal{U}_0 = q\Delta V}$$

- **An electron volt (eV)** is the kinetic energy acquired when a charge e is accelerated by a potential difference of 1 volt,

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.60 \times 10^{-19} \text{ J}$$

16.4 The electric potential

- Thus, with the conservation of energy principle ($K + \mathcal{U} = K_0 + \mathcal{U}_0$) and $\Delta\mathcal{U} = \mathcal{U} - \mathcal{U}_0 = q\Delta V$, then

$$\boxed{\Delta \boldsymbol{K} + \boldsymbol{q}\Delta \boldsymbol{V} = \mathbf{0} \quad \textbf{or} \quad (\boldsymbol{K} - \boldsymbol{K}_0) + \boldsymbol{q}\Delta \boldsymbol{V} = \mathbf{0}}$$

Example 16.4

In the cathode-ray tube of an oscilloscope or a television picture tube, electrons are accelerated from rest through a potential difference of +20,000 V. What is their velocity? (The electron mass is 9.11 $\times 10^{-31}\ kg$, and the charge is q $= -e = -1.6\ x\ 10^{-19}\ C$.)

Answer

$$(K - K_0) + q\Delta V = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) - e\Delta V = 0$$

$$v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{(2)(1.6\ x\ 10^{-19}\ C)(20000\ V)}{9.11\times10^{-31}\ kg}} = 8.38\times10^{7} m/s$$

16.4 The electric potential

Example 16.4

A proton is accelerated from rest through a potential difference of 5000 V. What its their velocity? (The electron mass is $1.67 \times 10^{-27}\ kg$, and the charge is q $= e = 1.6\ x\ 10^{-19}\ C$.)

Answer

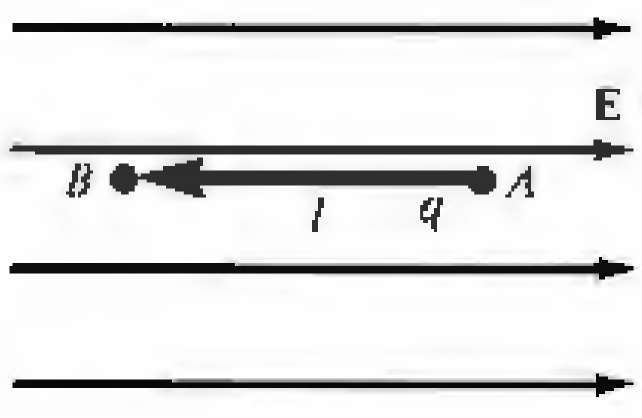
$$\Delta K + \Delta U = 0$$

$$(K - K_0) + q\Delta V = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + e\Delta V = 0$$

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{(2)(1.6\ x\ 10^{-19}\ C)(-5000\ V)}{1.67\times10^{-27}\ kg}} = 9.8 \times 10^5 m/s$$

16.4 The electric potential



- We can find a relationship between the electric potential and field by considering a positive charge q in a constant electric field $\boldsymbol{E}$.
- We suppose a force $\boldsymbol{F}$ equal but opposite to the electric force $q\boldsymbol{E}$ is applied so that the charge moves at a constant velocity from A to B.
- When the charge moves a distance l opposite to the field, the applied force does work

$$W = Fl = qEl$$

- Since the kinetic energy remains constant, this work must equal the change in the potential energy of the charge:

$$\boxed{\boldsymbol{W = \Delta\mathcal{U} = qEl}}$$

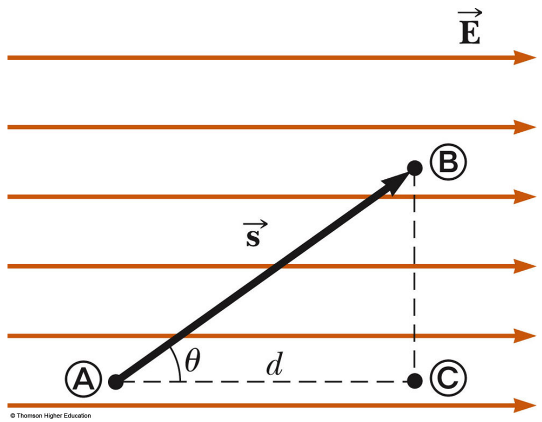
- In general, in a uniform electric field $\boldsymbol{E}$, the potential difference between two points displaced by a displacement $\boldsymbol{s}$ can be found as:

$$\Delta V = \frac{\Delta\mathcal{U}}{q} = \frac{-W_{\text{field}}}{q} = \frac{-q\boldsymbol{E}\cdot\boldsymbol{s}}{q} = -\boldsymbol{E}\cdot\boldsymbol{s} = -Es\cos\theta,$$

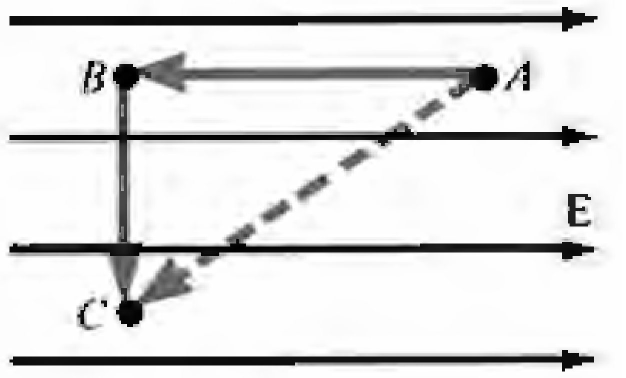
where W_{field} is the work don by the field and θ is the angle between $\boldsymbol{E}$ and $\boldsymbol{s}$

- Thus, in a uniform electric field, the potential difference between any two points is

$$\boxed{\Delta V = -\boldsymbol{E}\cdot\boldsymbol{s} = -Es\cos\theta}$$



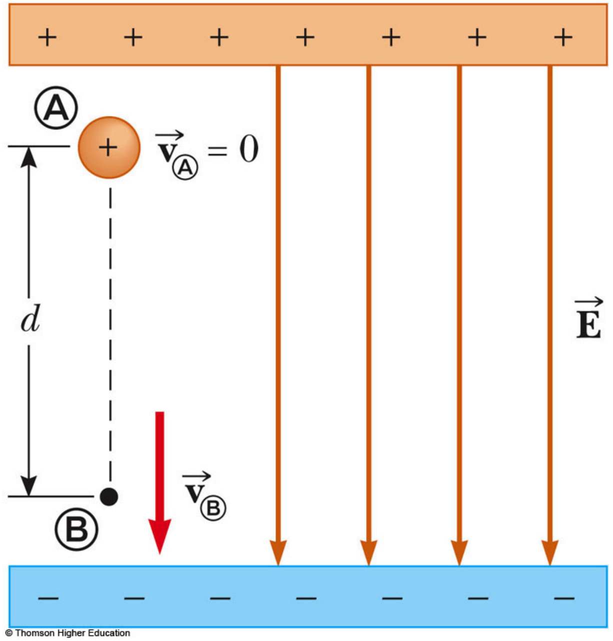
16.4 The electric potential



- Electric force is conservative. Thus, the work done by the conservative electric force is the same for the paths ABC and AC.
- The relation of the potential difference in a uniform electric field is
  $$\Delta V = -\boldsymbol{E} \cdot \boldsymbol{s} = -Es\cos\theta$$
  implies that
  - When $\boldsymbol{s}$ is parallel to $\boldsymbol{E}$ $(\theta = 0)$ then $\Delta V = -\boldsymbol{E} \cdot \boldsymbol{s} = -Ed$ decreases (negative).
    with $\Delta\mathcal{U} = q\Delta V$, then
    - The potential energy of a positive charge decreases when it is moved opposite to the field and gains the same amount as kinetic energy.
    - The potential energy of a negative charge increases when it is moved opposite to the field and loose the same amount of kinetic energy.
  - When $\boldsymbol{s}$ is opposite to $\boldsymbol{E}$ $(\theta = \pi)$ then $\Delta V = -\boldsymbol{E} \cdot \boldsymbol{s} = Ed$ increases (positive)
    with $\Delta\mathcal{U} = q\Delta V$, then
    - The potential energy of a positive charge increases when it is moved opposite to the field and loose the same amount of kinetic energy
    - The potential energy of a negative charge decreases when it is moved opposite to the field and gains the same amount as kinetic energy.
  - When $\boldsymbol{s}$ is normal to $\boldsymbol{E}$ $(\theta = \pi/2)$ then $\Delta V = 0$ (the potential is constant) and then $\Delta\mathcal{U} = 0$. This means that no work is done in moving a charge normal to $\boldsymbol{E}$.

16.4 The electric potential

Example:
A proton is released from rest at point A in a uniform electric field that has a magnitude of $8.0 \times 10^4\, V/m$. The proton undergoes a displacement of magnitude $d = 0.50\ m$ to point B in the direction of $\boldsymbol{E}$. Find the speed of the proton after completing the displacement.



Answer

$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}mv^2 - 0\right) + e\,\Delta V = 0$$

$$v = \sqrt{\frac{-2e\,\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \mathrm{C})(8.0 \times 10^4\ \mathrm{V})(0.50\ \mathrm{m})}{1.67 \times 10^{-27}\ \mathrm{kg}}}$$

$$= \boxed{2.8 \times 10^6\ \mathrm{m/s}}$$

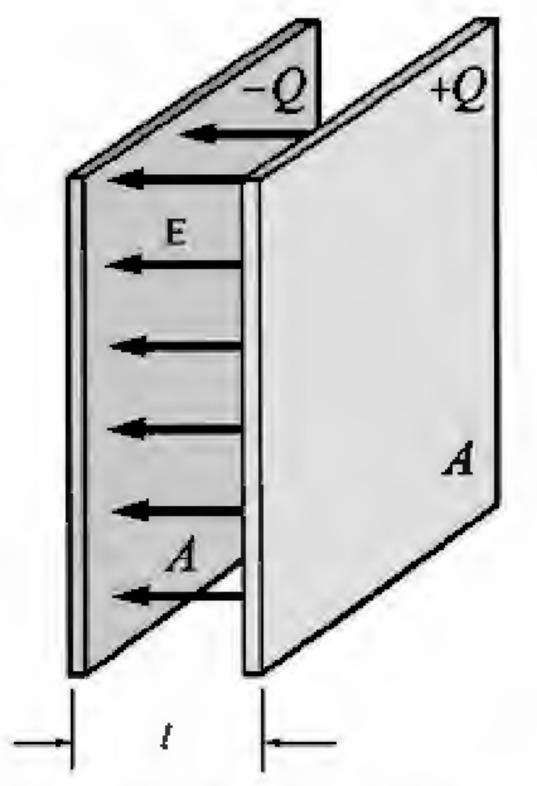
16.4 The electric potential

- The field between two oppositely charged plates is uniform.
- If the plate area is A and the charges are $+Q$ and $-Q$, the magnitude of the field is

$$E = \frac{4\pi kQ}{A}$$

.Thus, when the plates are separated by a distance l, the potential difference between them is

$$\Delta V = El = \frac{4\pi kQ}{A} l$$



Example 16.5

Two oppositely charged parallel plates have an area of 1 m^2 and are separated by 0.01 m. The potential difference between the plates is 100 V. Find (a) the field between the plates; (b) the magnitude of the charge on a plate.

Answer:

(a) Since E is uniform, then $\Delta V = El$ can be used

$$E = \frac{\Delta V}{l} = \frac{100\ \text{V}}{0.01\ \text{m}} = 10^4\ \text{V m}^{-1}$$

(b) The charge Q is

$$Q = \frac{A\,\Delta V}{4\pi k l} = \frac{(1\ \text{m}^2)(100\ \text{V})}{4\pi(9\times 10^9\ \text{N m}^2\ \text{C}^{-2})(10^{-2}\ \text{m})} = 8.84\times 10^{-8}\ \text{C}$$

16.4 The potential due to a point charge

- The electric potential at a distance r from a point charge Q is given by

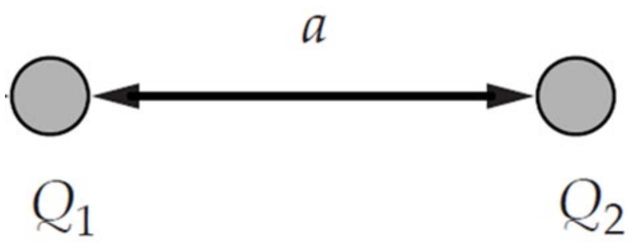
$$\boxed{V = \frac{kQ}{r} \qquad \text{(point charge)}}$$

- This expression implies that the electric potential has been chosen to be zero at $r = \infty$. We may do this since only potential differences can be measured, and we may define the potential to be zero at a convenient reference point, which in this case is at $r = \infty$.
- The electric potential for several point charges is just the sum of each individual charge.

$$\boxed{V = V_1 + V_2 + \cdots = \frac{kQ}{r_1} + \frac{kQ_2}{r_2} + \cdots = \sum_i \frac{kQ_i}{r_i}}$$

Example

If $a = 1.0\ m$, $Q_1 = 1.0\ nC$, and $Q_2 = -5\ nC$, What is the electric potential at the midpoint between the two charges

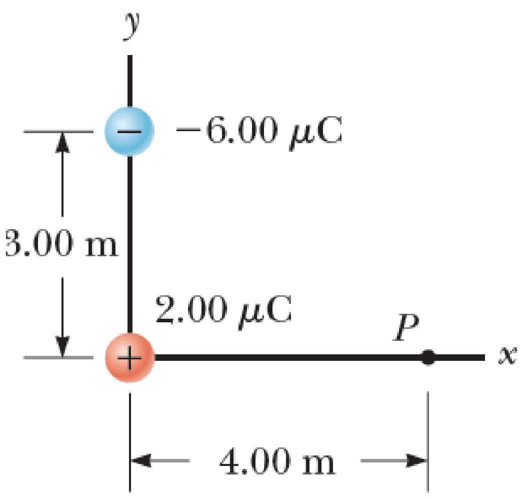


Answer:

$$V = k\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right) = (9 \times 10^9\ N.m^2.C^{-2})\left(\frac{1 \times 10^{-9}}{0.5^2} + \frac{-5 \times 10^{-9}}{0.5^2}\right) = -144\ V$$

16.4 The potential due to a point charge

Example:
As shown in Figure, a charge $q_1 = 2.0\ \mu C$ is located at the origin and a charge $q_1 = -6.0\ \mu C$ is located at $(0, 3.0)\ m$. (a) Find the total electric potential due to these charges at the point P, whose coordinates are $(4.0, 0)\ m$.
(b) What is the work required to bring a third charge from ∞ to point P.



Answer:

(a)

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (\ 9 \times 10^9\ \mathrm{N \cdot m^2/C^2})\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \mathrm{V}}$$

(b)

$$\mathrm{W} = \Delta U = U_f - U_i = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^3\ \mathrm{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \mathrm{J}}$$

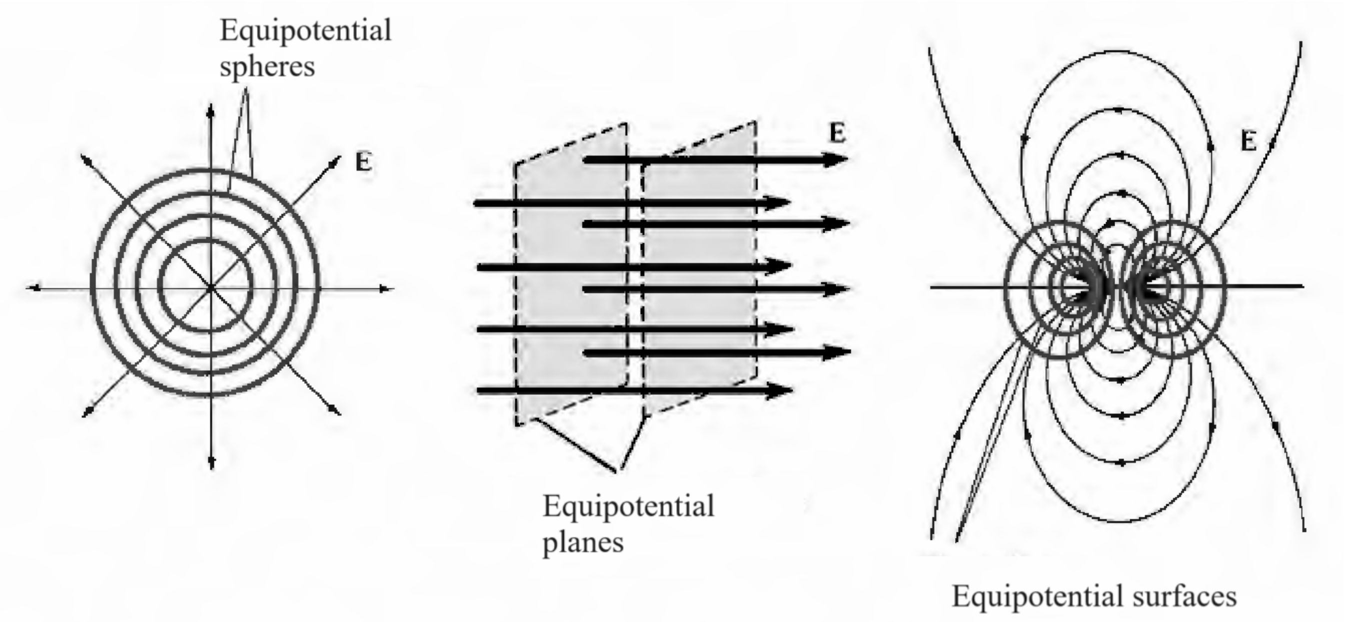
Equipotential surfaces

- **An equipotential surface is surface on which the potential is the same everywhere is called.**
  - For example, at any point on the surface of an imaginary sphere of radius r centered on a point charge, Q, the potential has the same value, $V = kQ/r$. Thus, for a point charge the equipotential surfaces are concentric spheres.
  - The equipotential surfaces for a uniform electric field are planes normal to the field.

- When a charge moves at right angles to the electric field, no work is done against electrical forces, so its potential energy remains constant.

- For this reason, **the equipotential surfaces are always perpendicular to the electric field lines.**

- Charges may move along an equipotential surface with no change in potential energy.



16.8 Capacitance

- A **capacitor** is an arrangement of two conductors (called plates) separated by a an insulator.
- When a potential difference is applied across the plates, small amounts of charge is taken from one plate and placed on the other.
- When the capacitor is charged, the plates carry charges of equal magnitude and opposite sign
- **The capacitance (C) of a capacitor is defined as ratio of the amount of charge transferred to the potential difference**.
- The energy required to separate the charges is stored in the capacitor. Hence the capacitance is also a measure of the ability to store energy.
- If two conductors have equal and opposite charges $\pm Q$ and a corresponding p

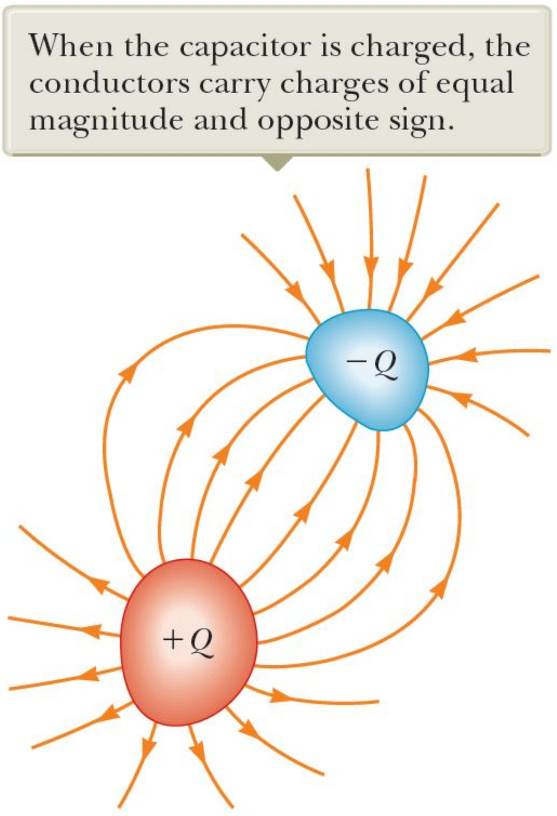
  the ratio $\frac{Q}{V}$ is usually found to be a constant independent of Q. The ratio is the capacitance

$$\boxed{C = \frac{Q}{V}}$$

The unit of capacitance is the farad (F); $1\ F = 1\ C/V$.

- Because the coulomb is a large unit, the farad is also large, and most capacitors have small values in terms of the farad. Hence, we define the microfarad and picofarad by

$$1\ \mu\text{F} = 10^{-6}\ \text{F}, \qquad 1\ \text{pF} = 10^{-12}\ \text{F}$$



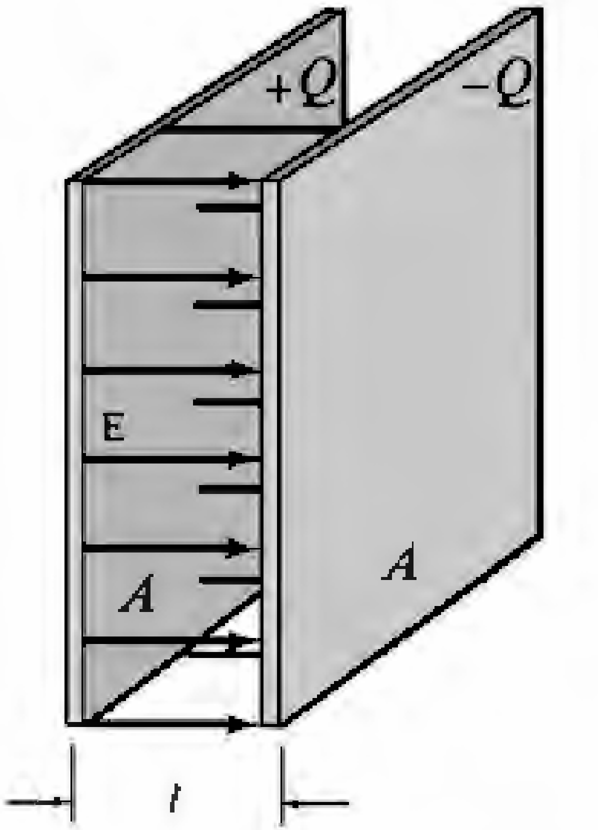
The parallel plate capacitor

- The simplest capacitor is composed of two parallel plates in a vacuum.
- The plates have surface area A, charges $+Q$, and a separation d.
- The capacitance of the parallel plate capacitor is given by

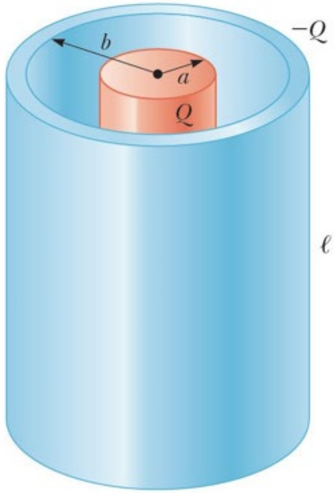
$$\boxed{C = \frac{A}{4\pi k l} = \frac{\varepsilon_0 A}{l}}$$

where

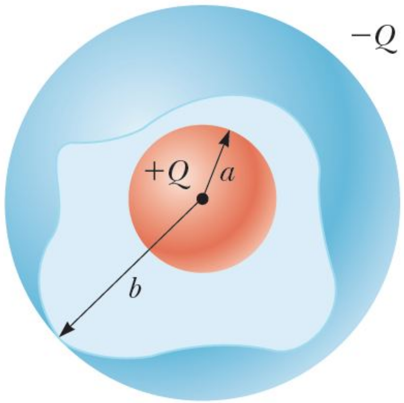
$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$$



- Other kinds of capacitors are cylindrical and spherical capacitors



cylindrical capacitor



spherical capacitor

The parallel plate capacitor

Example 16.10

A $1\ pF$ capacitor is connected to a $12\ V$ battery. What are the charges on its plates?

Answer

From the definition of the capacitance, $C = Q/V$, we have

$$Q = CV = (10^{-6}\ \mathrm{F})(12\ V) = 1.2 \times 10^{-5}\ \mathrm{C}$$

Energy stored in a capacitor

- A charged capacitor stores electrical energy.
- If its plates are connected by conducting wire, electrons will move in the wire from the negative plate to the positive plate.
- Thus, stored electrical energy can be transformed into other forms of energy.
- The energy stored in a capacitor may initially be supplied by a battery, which maintains a potential difference between its two terminals. When the plates of an uncharged capacitor are connected by two wires to the terminals, electrons flow in the wires from one plate through the battery to the other until the potential difference across the capacitor reaches a maximum value. We can find the energy stored in the capacitor by calculating the work that must be done by the battery in building up this charge from zero to the final value Q.
- The energy stored in a capacitor of capacitance C and of charge Q and potential difference V is given by

$$\boxed{\mathcal{U} = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2}$$

Example 16. 13

One square centimeter of membrane has a capacitance of $7.08\ x\ 10^{-7}\ F$. If the potential difference across the membrane is $0.1\ V$, find the electrical energy stored in $1\ cm^2$ of membrane.

Answer

Since we know C and V, we write the energy as

$$\begin{aligned}\mathcal{U} &= \tfrac{1}{2}CV^2 = \tfrac{1}{2}(7.08 \times 10^{-7}\ \text{F})(0.1\ \text{V})^2 \\ &= 3.54 \times 10^{-9}\ \text{J}\end{aligned}$$

Energy stored in a capacitor

Example 16.14 The separation between the plates of a parallel plate capacitor is doubled while the charge on them is held constant. What happens to the (a) electric field between the plates; (b) potential difference; (c) capacitance; (d) stored energy?

Answer

(a) The electric field between two oppositely charged parallel plates is $4\pi kQ/A$. Since neither the charge nor the plate area changes when the plate separation is increased, the field, $\boldsymbol{E}$, is unchanged.

(b) The potential difference between the plates is the work per unit charge needed to move charge from one plate to the other, or $V = El$. Since l has been doubled, V is doubled.

(c) The capacitance is defined as the ratio $C = Q/V$. Here Q is constant and V is doubled, so C is halved.

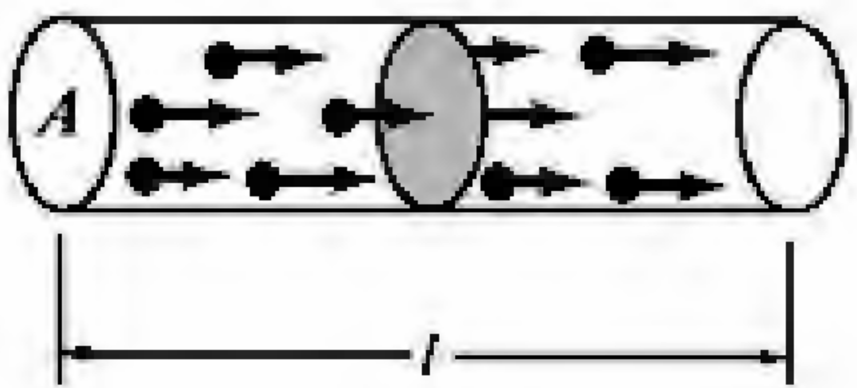
(d) The stored energy can be written as $\mathcal{U} = \frac{1}{2}QV$. Since Q is constant and V is doubled, the energy $\mathcal{U}$, is doubled. The two plates are oppositely charged and attract each other. The mechanical work we do against this force when we increase their separation is the source of the increased energy stored by the capacitor

17.1 Electric current

- **The electric current** in a wire is the rate at which charge moves in the wire.
- In the figure, charges move through a conducting wire under the influence of an applied electric field.
- If a net charge ΔQ crosses the shaded cross-sectional area in a time Δt, **the average current** is

$$\boxed{\bar{I} = \frac{\Delta Q}{\Delta t}}$$

the average current



- The **instantaneous current** is

$$\boxed{I = \frac{dQ}{dt}}$$

instantaneous current

- The S.I. current unit is the ampere (A).
- From the definition, it follows that an ampere is a coulomb per second C/s.
- Often it is convenient to use the milliampere (mA); $1\ mA = 10^{-3} A$.

17.1 Electric current

Example 17.1

An electrochemical cell consists of two silver electrodes placed in an aqueous solution of silver nitrate. A constant 0.5-A current is passed through the cell for I hour. Find the total charge transported through the cell in coulombs and in multiples of the electronic charge.

Answer

Since the current is constant

$$\Delta Q = I\,\Delta t = (0.5\ \mathrm{A})(1\ \mathrm{h}) = (0.5\ \mathrm{C\ s^{-1}})(3600\ \mathrm{s}) = 1800\ \mathrm{C}$$

The ratio of ΔQ to the electronic charge is

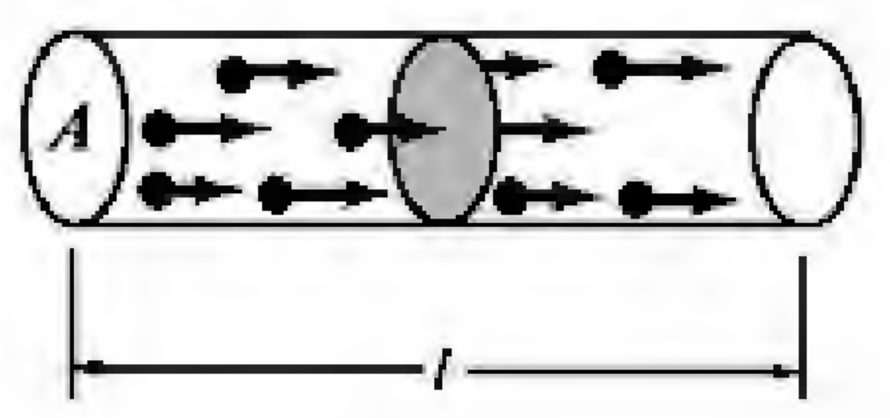
$$N = \frac{\Delta Q}{e} = \frac{1800\ \mathrm{C}}{1.60 \times 10^{-19}\ \mathrm{C}} = 1.13 \times 10^{22}$$

This is the number of silver ions transported through the cell and deposited in 1 hour.

17.1 Electric current

- Conventionally the current in a conductor is assumed to be **in the direction of motion of positive charges** (conventional current). However, in metallic conductors the moving charges are electrons.
- We can relate the current in a wire to the density of conduction electrons and their drift velocity v.
- If there are n electrons per unit volume, then the total number of electrons in a volume V is nV. The segment of wire in the figure has length l and cross-sectional area A, so its volume is $V = lA$.
- Hence there are $\mathrm{N} = nV = nlA$ electrons in the wire.
- The total charge has a magnitude $\mathrm{e}N = enV = enlA$.
- The time needed for all of them to pass through the end of the segment shown is $\Delta t = l/v$,
- So the magnitude of the current is

$$\boxed{I = \frac{\Delta Q}{\Delta t} = \frac{en\ell A}{\ell/v} = enAv}$$



17.1 Electric current

Example 17.2

Number 12 copper wire is often used to wire house hold electrical outlets. Its radius is $1\,mm = 10^{-3}\,m$. If it carries a current of $10\,A$, what is the drift velocity of the electrons? (Metallic copper has one conduction electron per atom, the atomic mass of copper is $64u$, and the density of copper is $8900\,kg.m^3$.)

Answer

The number of atoms per unit volume times the mass of one atom M equals the mass of a unit volume of copper, which is its density d. Thus $nM = d$

$$n = \frac{d}{M} = \frac{8900 \text{ kg m}^{-3}}{(64 \text{ u})(1.66 \times 10^{-27} \text{ kg u}^{-1})}$$

$$= 8.38 \times 10^{28} \text{ m}^{-3}$$

The drift velocity is then

$$v = \frac{I}{neA} = \frac{I}{ne\pi r^2}$$

$$= \frac{(10 \text{ A})}{(8{,}38 \times 10^{28} \text{ m}^{-3})(1{,}6 \times 10^{-19} \text{ C})\pi(10^{-3} \text{ m})^2}$$

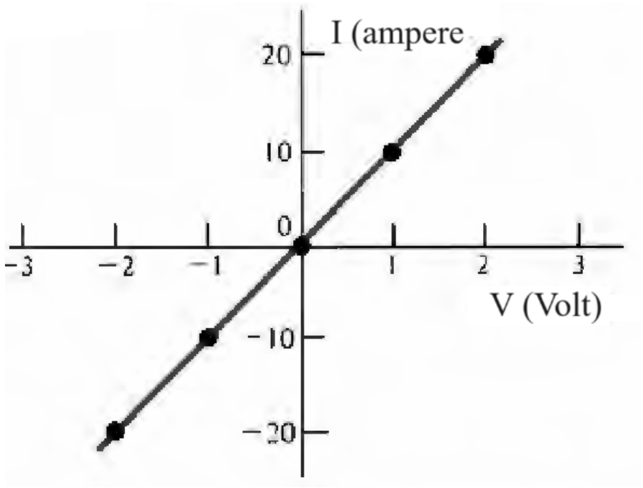
$$= 2{,}37 \times 10^{-4} \text{ m s}^{-1}$$

17.2 Resistance

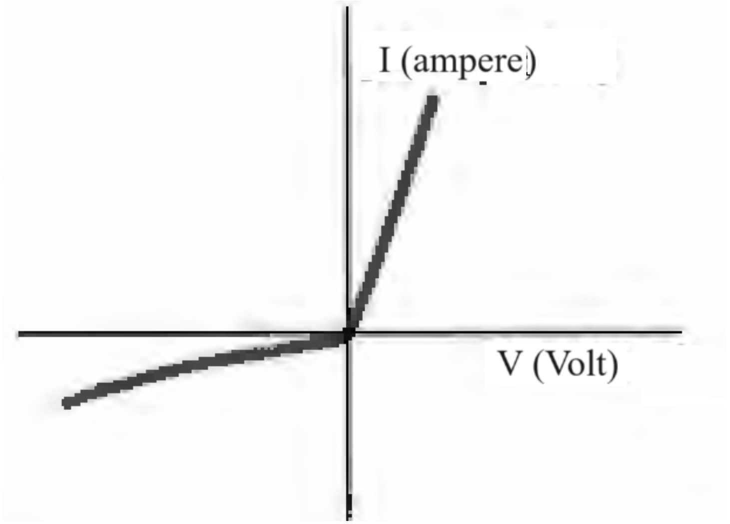
- The electrical resistance R of a conductor is the potential difference V between current I,

$$\boxed{R = \frac{V}{I}}$$

- The S.I. unit for resistance is the ohm; an ohm is a volt per ampere.
- For many materials, the potential difference and the current are directly proportional, so the resistance is a constant independent of the current.
- Materials with a constant resistance are said to obey Ohm's law and are called **ohmic conductors**.
- The resistance of some conductors varies with the magnitude or direction of the applied potential difference. These materials are called **nonohmic materials**



ohmic conductors

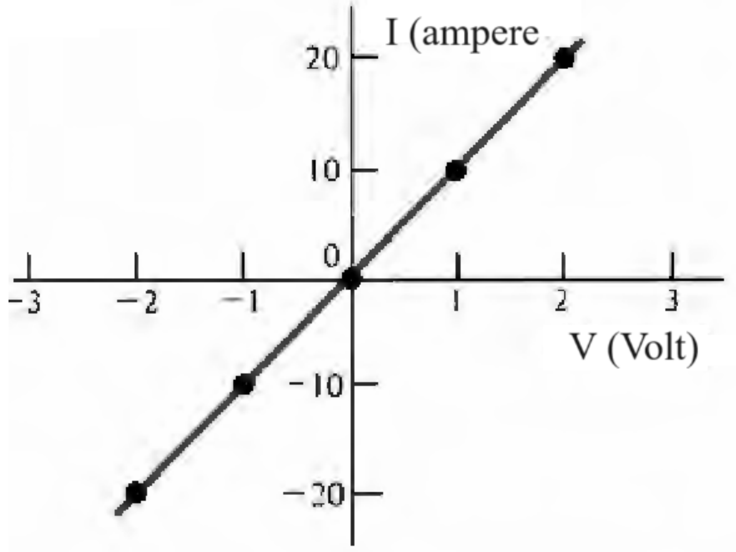


nonohmic conductors

17.2 Resistance

Example 17.3

Find the resistance of the wire in the figure



Answer

The potential difference and current are proportional, so the resistance is a constant. When the current is $10\ A$, the potential difference is $1\ V$, and

$$R = \frac{V}{I} = \frac{1\text{ V}}{10\text{ A}} = 0.1\text{ ohm}$$

- The resistance of a conductor depends on its size, shape, and composition.
- The resistance in terms of geometric factors and a constant can be written as

$$\boxed{R = \frac{\rho \ell}{A}}$$

- The proportionality constant ρ depends only on the properties of the material and is called the **resistivity**.
- The S.I. unit for resistivity is the ohm meter.
- The conductivity σ, is defined by

$$\boxed{\sigma = \frac{1}{\rho}}$$

17.2 Resistance

Example

A copper wire 100 m long with a radius of $1\ mm = 10^{-3}\ m$. (a) Find the room temperature resistance of the wire. (b) If a potential difference of 3 V is applied across the ends of the wire, what is the current passing the wire.

Answer

(a) According to Table 17.1, the resistivity of copper at room temperature (20 °C) is $1.72\ \times 10^{-8}\ ohm\ m$.

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2}$$

$$= \frac{(1.72 \times 10^{-8}\ \text{ohm m})(100\ \text{m})}{\pi (10^{-3}\ \text{m})^2}$$

$$= 0.547\ \text{ohm}$$

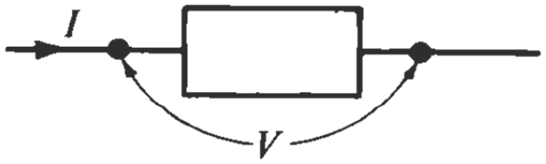
(b) The current is obtained using Ohm's law:

$$I = \frac{V}{R} = \frac{3\ V}{0.547\ ohm} = 5.48\ A$$

17.4 Power in electrical circuits

- In a circuit, energy is initially converted from some other form by a battery or a generator into electrical potential energy. It is then transformed in the load into heat, mechanical work, or some other kind of energy
- The power dissipated in a resistance R when a current I passing through the resistance and potential difference V is applied across its terminals can be found using $\mathcal{P} = \frac{\Delta W}{\Delta t}$

$$\boxed{\mathcal{P} = IV = I^2R = \frac{V^2}{R}}$$



Example 17.9

For a household $60\ W$ light bulb operated at $120\ V$, find (a) the current; (b) the resistance; (c) the 24-hour operating cost, if energy costs 10 cents per kilowatt hour

Answer

(a) Since the power $\mathcal{P}$ in any circuit element equals VI:

$$I = \frac{\mathcal{P}}{V} = \frac{60\ \mathrm{W}}{120\ \mathrm{V}} = 0.5\ \mathrm{A}$$

(b) The resistance is

$$R = \frac{V}{I} = \frac{120\ \mathrm{V}}{0.5\ \mathrm{A}} = 240\ \mathrm{ohms}$$

(c) The bulb uses power at the rate of 60 W or $0.060\ kW$. Thus the cost of operating it for 24 hours is

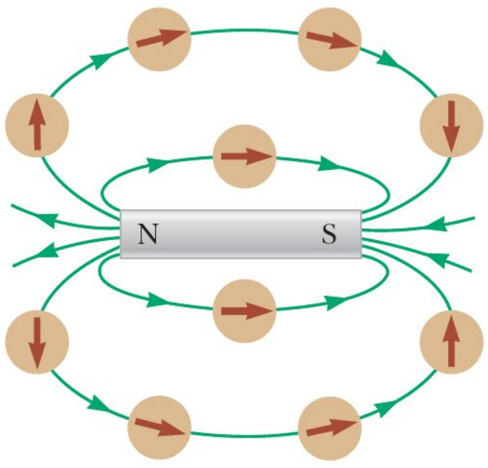
$$(0.06\ \mathrm{kW})(24\ \mathrm{h})[10¢\ (\mathrm{kW\ h})^{-1}] = 14.4¢$$

17.8 Electrical safety

- Electrical equipment must be designed and used with care to avoid possible fire and electrocution hazards.
- Each circuit in a building is protected by a fuse or a circuit breaker and a ground line

- Electrocution is a serious hazard, since relatively small currents through the human torso can cause injury or death. The average adult can detect a current as small as $1\ mA = 10^{-3}\ A$
  - Relatively small current passing through the human torso can cause injury or death.
  - The maximum allowable current is less than 1 mA.
  - Few milliamperes cause muscular reaction and pain
  - Between 10 mA and 20 mA will paralyze muscles and prevent a person from releasing a conductor.
  - About 18 mA contracts the chest muscles and causes breathing to stop.
  - Unconsciousness and death will occur in minutes unless the current stop
  - 100 mA for few seconds stop the heart muscles from pumping blood

19.1 Magnetic field



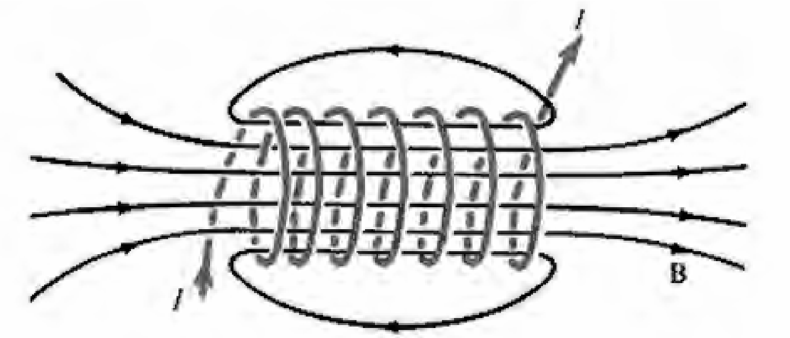
- Every magnet, regardless of its shape, has two poles
  - Called north (N) and south poles (S)
  - Poles exert forces on one another
    - Like poles repel each other N-N or S-S
    - Unlike poles attract each other N-S
  - A single magnetic pole has never been isolated. Therefore, magnetic poles are always found in pairs.

- Magnetic field ($\boldsymbol{B}$): The region of space surrounding
  - currents (moving charges)
  - magnetic materials

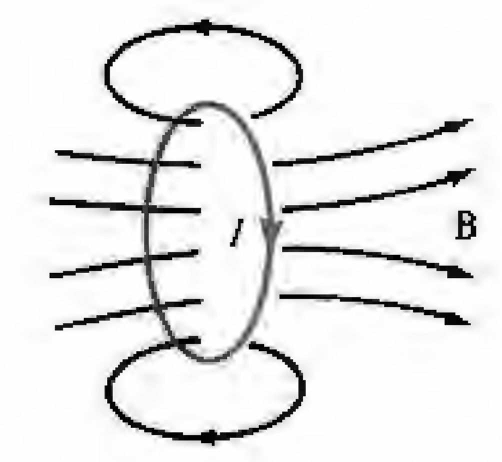
- Magnetic fields are represented by diagrams.
  - By convention, the magnetic field lines are always directed from the north pole to the south pole outside a magnet.
  - Direction of the magnetic field is tangent to the line.
  - Density of the lines is proportional to the magnitude of the magnitude of the magnetic field.
  - The field lines form a closed loop.
  - The field lines never cross

19.1 Magnetic field

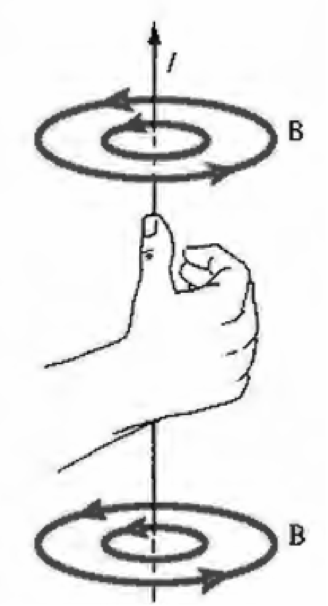
- Electric currents also produce magnetic fields:



Magnetic field lines due to a current in a long coil of wire. Note the similarities to the field lines and patterns for the long bar magnet

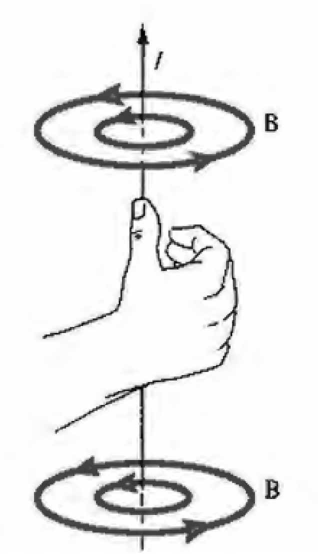


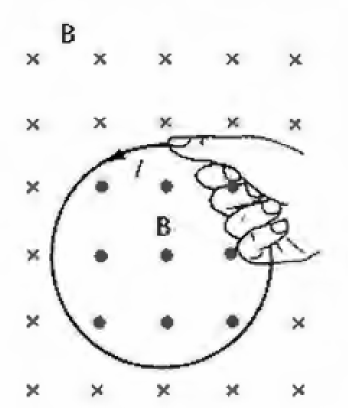
Magnetic field lines due to a single current turn



Magnetic field lines (circular lines) due to a long straight wire carrying current I

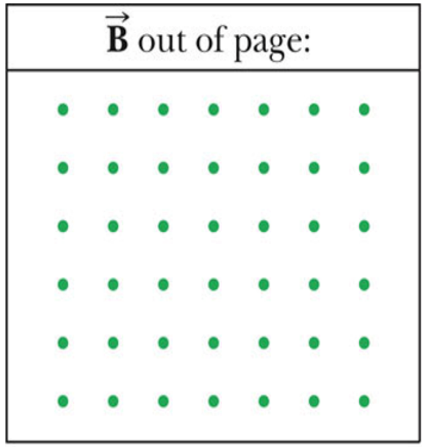
- **The Right-Hand Rule** is used to relate the direction of the current and the magnetic field **B**:
    - If the right thumb is placed along the wire in the direction of the current, the fingers curl in the direction of the field. Similarly, for the circular current loop.

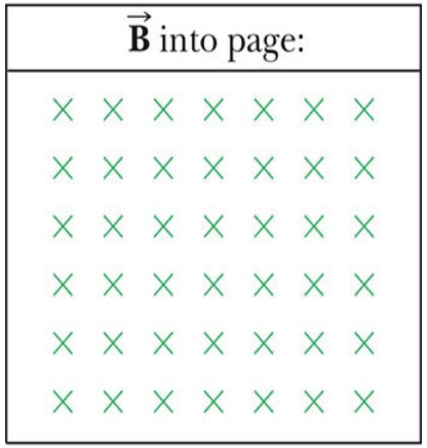




19.2 Magnetic field

- When vectors are perpendicular to the page, dots and crosses are used
  - The dots represent the arrows coming out of the page
  - The crosses represent the arrows going into the page





- The S.I. magnetic field unit, the tesla (T), is a rather large unit.
  - The largest fields produced in the laboratory for an extended time are about $10\ T$;
  - fields up to about 100 T can be produced very briefly.
  - The earth's magnetic field at the surface of the earth is about $10^{-4}\ T$.

19.2 The magnetic force on a moving charge

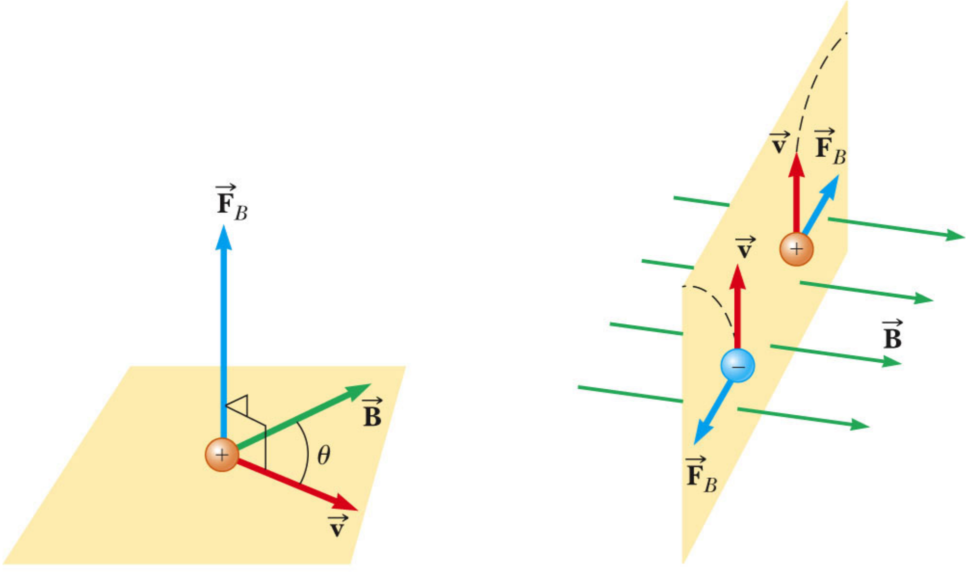
- Electric charges in motion near a magnet experience forces that can readily be demonstrated,
- A positive charge q moving with velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{B}$ experiences a magnetic force $\boldsymbol{F}$ perpendicular to $\boldsymbol{v}$ and to $\boldsymbol{B}$. The magnitude of this force is the product of q, v, and the component of $\boldsymbol{B}$ perpendicular to $\boldsymbol{v}$.
- Using the cross-product notation, the magnetic force $\boldsymbol{F}$ is given

$$\boxed{\mathbf{F} = q\mathbf{v} \times \mathbf{B}}$$

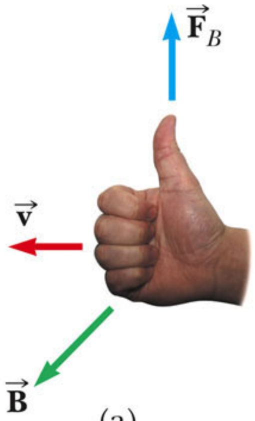
- The magnitude of this force is

$$\boxed{F = |q|vB \sin\theta}$$

- The force on a negative charge is opposite to the force on the positive charge.
- Right hand rule is used to determine the direction of $\boldsymbol{F}$



(a) (b)

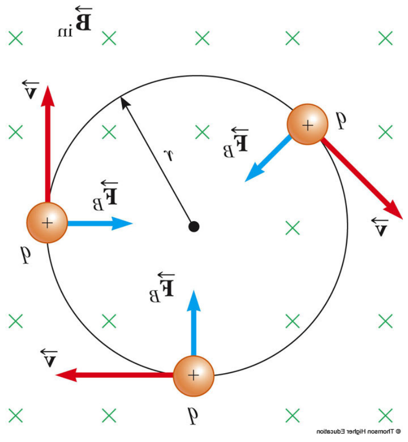


(a)

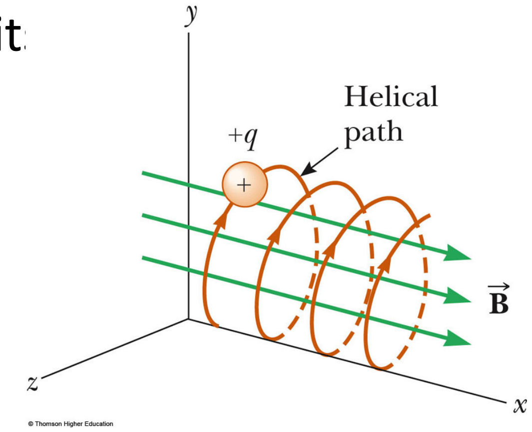
19.2 The magnetic force on a moving charge

- With $\boldsymbol{F} = |q|vBsin\theta$:
  - If $\theta = 0$ or π then $\boldsymbol{F} = 0$ and thus the magnetic field does not affect the motion of the charge.

- If $\theta = \pi/2$ the charged particle will move in a circular path



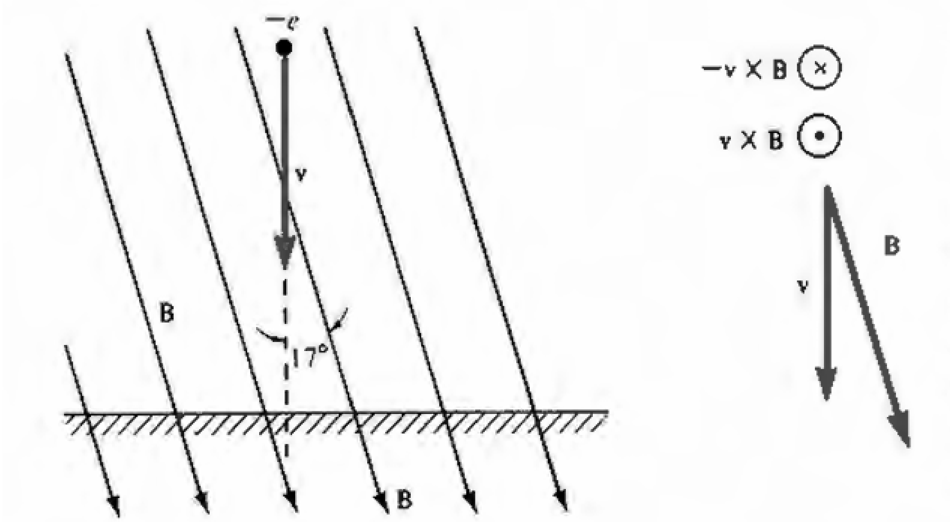
- If a charged particle moves in a magnetic field at some arbitrary angle with respect to the field, it: path is a helix



19.2 The magnetic force on a moving charge

Example 19.1

At Boston, Massachusetts, the magnetic field due to the earth is at $17°$ to the vertical direction and has a magnitude of $5.8 \times 10^{-5}\ T$ as shown in the figure. (a) Find the force $\boldsymbol{F}$ on an electron moving straight down at $10^5\ m/s$. (b) Find the ratio of $\boldsymbol{F}$ to the weight mg.



Answer

(a) With $q = -e = -1.6 \times 10^{-19}$ C and $\sin 17° = 0.292$, the magnitude of the magnetic force is

$$F = evB \sin\theta$$
$$= (1.6 \times 10^{-19}\ \text{C})(10^5\ \text{m s}^{-1})(5.8 \times 10^{-5}\ \text{T})(0.292)$$
$$= 2.71 \times 10^{-19}\ \text{N}$$

Since $\mathbf{F} = -e\mathbf{v} \times \mathbf{B}$, the force is opposite to $\mathbf{v} \times \mathbf{B}$ or into the page in Fig. 19.14.

(b) The mass of an electron is 9.1×10^{-31} kg, so

$$\frac{F}{mg} = \frac{2.71 \times 10^{-19}\ \text{N}}{(9.1 \times 10^{-31}\ \text{kg})(9.8\ \text{m s}^{-2})}$$
$$= 3.04 \times 10^{10}$$

19.2 The magnetic force on a moving charge

- The electric current can be produced by the magnetic field (electric generator)

- A closed ring rotating in a magnetic field generates electric field

  - The reason for that is that the magnetic force will induce the free charges in the ring to move across it.