Chapter 12
Thermal properties of matter

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COURSE TOPICS:

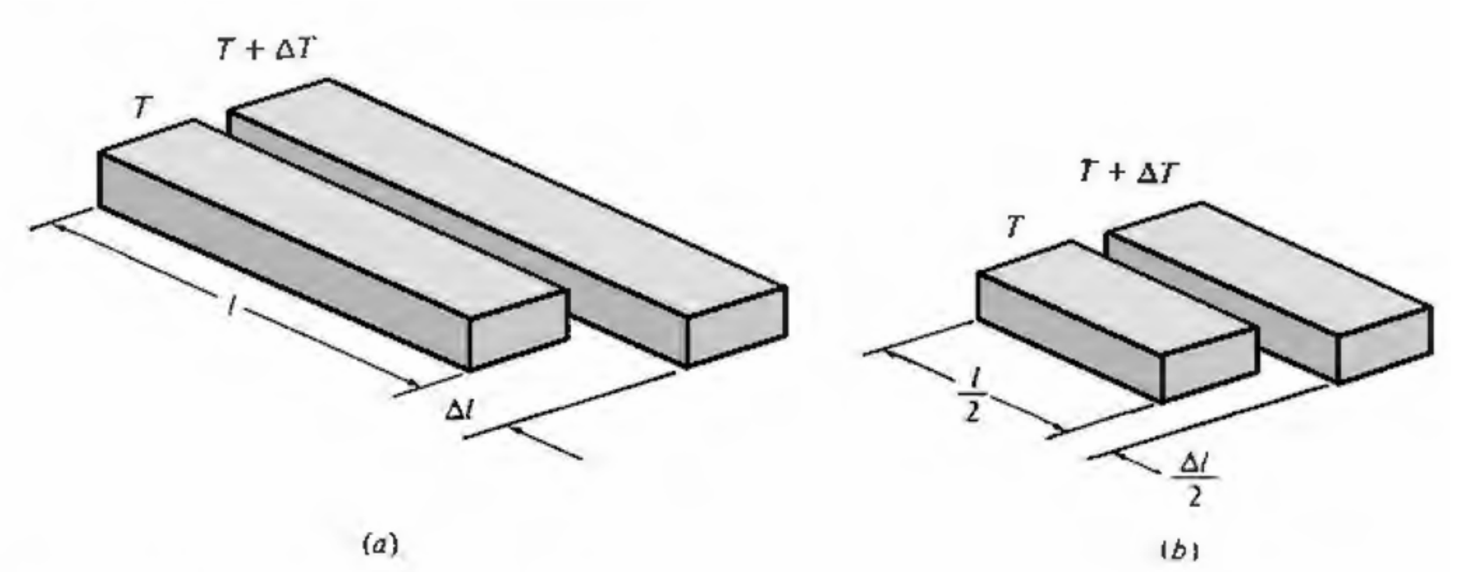
12.1 Thermal expansion
12.2 Heat capacity
12.3 Phase changes

Introduction:

- Because the properties of matter depend on temperature, the exchange of thermal energy is extremely important.
- Since biological process can only function properly over a small temperature range, both people and nature have devised ways of either limiting or improving the means by which this energy is transferred.
  - For example, we insulate our homes, and our bodies respond to high temperatures by perspiring.
- We begin our study of the thermal properties of matter with the thermal expansion of solids and liquids.
- We then discuss how the temperature of an object changes as thermal energy or heat is added or withdrawn.

12.1 Thermal expansion:
Linear expansion

- When a substance is heated, its volume usually increases, and each dimension increases correspondingly.
- This increase in size can be understood in terms of the increased kinetic energy of the atoms or molecules.
    - The additional kinetic energy results in each molecule colliding more forcefully with its neighbors. The molecules effectively push each other farther apart, and the material expands.
- At the macroscopic level, we can find a convenient relation between the change in length of an object and the temperature change:
    - Suppose the original length of the object in the figure is l.
    - a small increase in length Δl occurs when the temperature increases by a small amount ΔT.
    - If we divide the object into two equal parts, each part will have length $l/2$ and will expand by $\Delta l/2$.
    - Consequently, the change in length Δl is directly proportional to the length l
    - In addition, we find from experiment that if we double the temperature change by raising the temperature by $2\ \Delta T$, the expansion also doubles.



12.1 Thermal expansion:
Linear expansion

- A single equation expressing both proportionalities is

$$\Delta l = \alpha l \, \Delta T$$

- The constant α is the coefficient of linear expansion.
- It is a property of a given material and depends somewhat on the temperature.
- α has the dimensions of inverse temperature, so its units are K^{-1} .
- Since only the change in temperature is important, we can measure ΔT in Kelvins or in Celsius degrees.
- Since α depends somewhat on temperature, the above equation is exact only for very small temperature changes

TABLE 12.1

Coefficient of linear expansion for various materials

Material	Temperature (°C)	α (K^{-1})
Aluminum	−23	2.21×10^{-5}
	20	2.30×10^{-5}
	77	2.41×10^{-5}
	527	3.35×10^{-5}
Diamond	20	1.00×10^{-6}
Celluloid	50	1.09×10^{-4}
Glass (most types)	50	8.3×10^{-6}
Glass (Pyrex)	50	3.2×10^{-6}
Ice	−5	5.07×10^{-5}
Steel	20	1.27×10^{-5}
Platinum	20	8.9×10^{-6}

12.1 Thermal expansion:
Linear expansion

Example 12.1

The roadbed of the Golden Gate Bridge is 1280 m long. During a certain year, the temperature varies from −12 ℃ to 38 ℃. What is the difference in the lengths at those temperatures if the road is supported by steel girders? For steel, $\alpha = 1.27\ x\ l0^{-5}\ K^{-1}$.

Answer

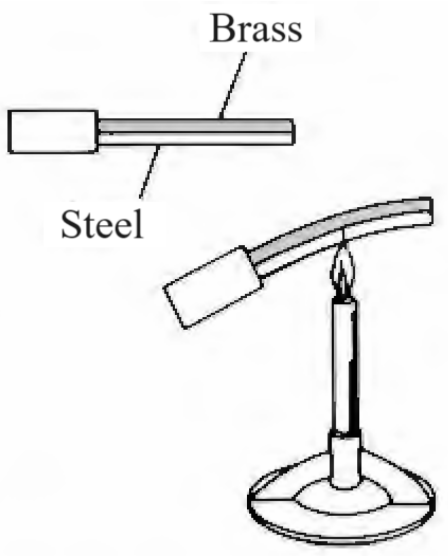
With $\Delta T = 38°\,C - (-12°\,C) = 50°\,C = 50\,K$,

$$\begin{aligned}\Delta l &= \alpha l\,\Delta T \\ &= (1.27 \times 10^{-5}\,K^{-1})(1280\,m)(50\,K) \\ &= 0.81\,m\end{aligned}$$

This substantial change in the roadbed length must be allowed for in the design of the bridge.

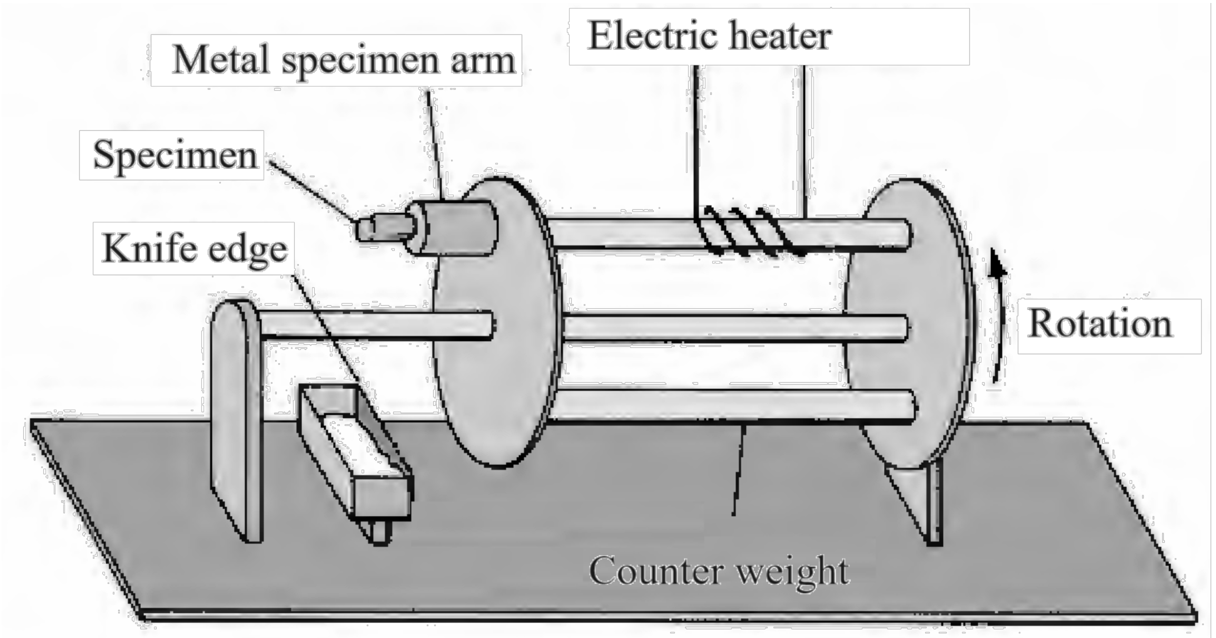
12.1 Thermal expansion:
Linear expansion

- Two interesting applications of thermal expansion are the thermostat and the ultramicrotome:
  - A **thermostat** has two metal strips with different coefficients of linear expansion attached to each other (Fig. 12.2). When they are heated the unequal expansion causes the strips to bend; if they bend far enough, they open or close a switch that may, for example, control a heating system or air conditioning.



thermostat

- The **ultramicrotome** is an apparatus designed to make very thin tissue slices for use in microscopes. A sample, mounted on a rotating metal arm, passes a knife edge as in the figure. If the metal arm is heated at a constant rate, it will expand uniformly, and a thin specimen slice will be cut on each turn. The metal arm can be extended as slowly as 1 micrometer (10^{-6} m) per minute.



ultramicrotome

12.1 Thermal expansion:
Area expansion

- Consider what happens to a square surface of a cube with an initial area l^2.
- An increase in temperature ΔT results in each side increasing in length to $l + \Delta l$.
- The new area is then

$$(l + \Delta l)^2 = l^2 + 2l\,\Delta l + (\Delta l)^2$$

- the increase in area is:

$$\Delta A = (l + \Delta l)^2 - l^2 = 2l\,\Delta l + (\Delta l)^2$$

- In general, Δl is very small compared with the original length l so we can neglect $(\Delta l)^2$ and make the approximation that

$$\Delta A \simeq 2l\,\Delta l = 2l(\alpha l\,\Delta T)$$

- Since l^2 was the initial area A, the change in area becomes

$$\boxed{\Delta A = 2\alpha A\,\Delta T}$$

- Thus, the area increases at twice the rate of a linear dimension.

12.1 Thermal expansion:
Area expansion

Example 12.2

A circular steel disk has a circular hole through its center. If the disk is heated from $10\ °C$ to $100\ °C$, what is the fractional increase in the area of the hole?

Answer

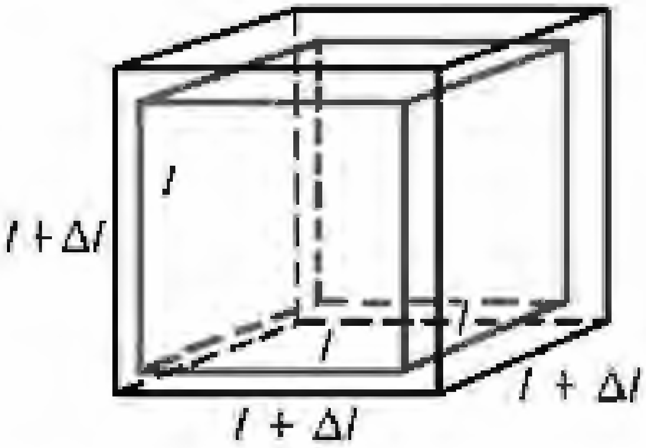
When the disk is heated, all of its dimensions in crease. The area of the hole increases also, just as the area inside a penciled circle on a solid disk increases. With $\alpha = 1.27 \times 10^{-5}\ K^{-1}$ for steel.

$$\Delta A = 2\alpha A\ \Delta T$$

$$\frac{\Delta A}{A} = 2\alpha\ \Delta T = 2(1.27 \times 10^{-5}\ \mathrm{K}^{-1})(90\ \mathrm{K})$$

$$= 2.29 \times 10^{-3}$$

12.1 Thermal expansion:
Volume expansion



- When all of the dimensions of an object increase, the volume also increases.
- This change in volume ΔV is written as

$$\Delta V = \beta V \, \Delta T$$

- Here β is the coefficient of volume expansion.
- β can be related to α by considering the volume change of a cube when the temperature is changed from T to $T + \Delta T$
- If the sides change from l to $l + \Delta l$, the change in volume is: $\Delta V = (l + \Delta l)^3 - l^3$
- The change in length Δl is much less than the original length.
- If $(l + \Delta l)^3$ is multiplied out, it is a very good approximation to keep only the first two terms, which are $l^3 + 3l^2\Delta l$
- Then the change in volume is just $\Delta V = 3l^2 \, \Delta l = 3V \, \Delta l / l.$
- Using $\Delta l = \alpha l \Delta T$, we have

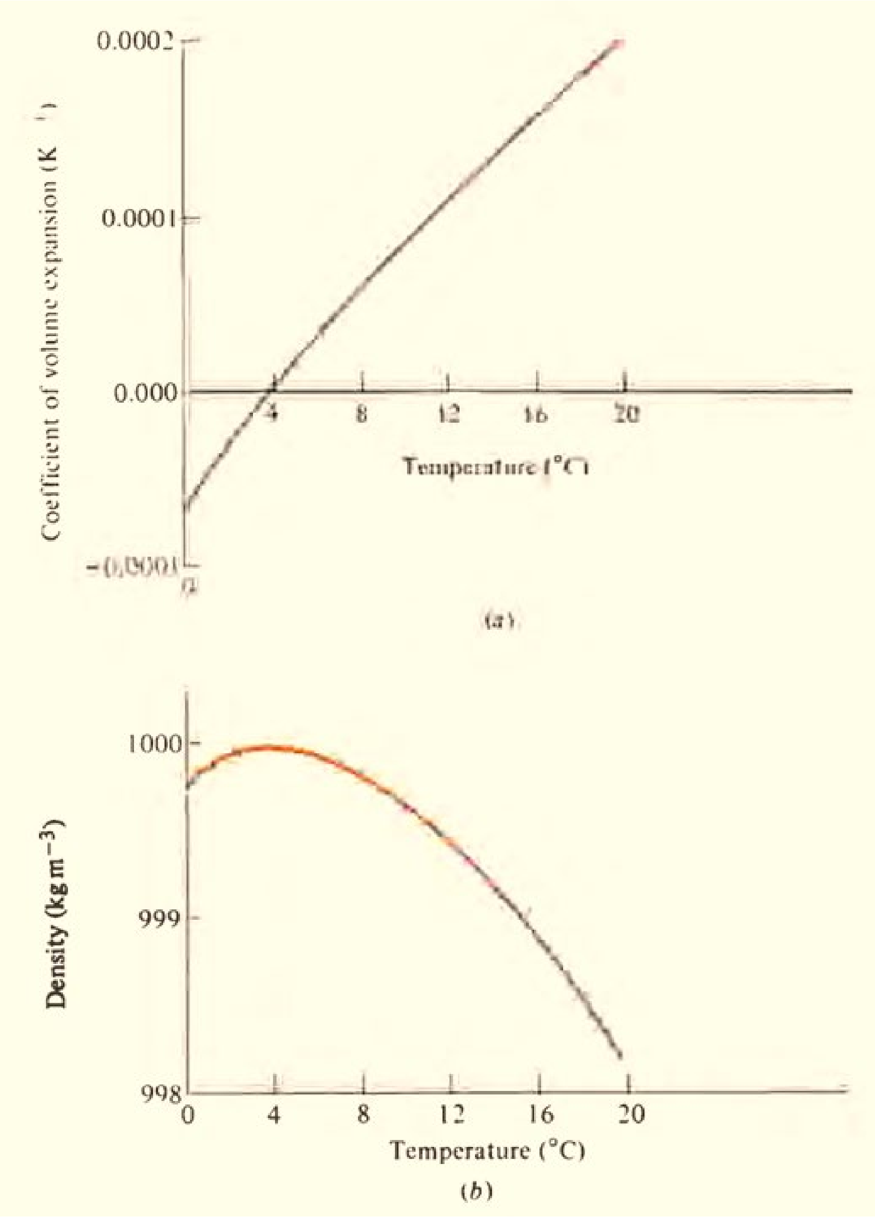
$$\Delta V = 3\alpha V \, \Delta T$$

- Comparing this with $\Delta V = \beta V \Delta T$, we see that

$$\beta = 3\alpha$$

12.1 Thermal expansion:
Water and negative coefficient of volume expansion

- Special mention must be made of liquid water because it is one of the very few substances with a negative coefficient of volume expansion at some temperatures.
- The figure shows both β and the density (mass per unit volume) of water plotted against the temperature.
- β varies as the temperature changes and even reverses sign at $3.98\ °C$.
- Thus, as T rises from $0\ °C$, water contracts up to $3.98\ °C$ and then expands as the temperature increases further;
- Water has its greatest mass per unit volume at $3.98\ °C$.
- This characteristic of water is extremely important for aquatic life. As the air temperature decreases in early winter, the surface water of lakes cools. When this surface water reaches $3.98\ °C$, it sinks to the bottom; the warmer, less dense water from beneath floats to the surface. The cool descending water carries oxygen with it. Once the entire lake has undergone this mixing and has reached $3.98\ °C$, further cooling occurs at the surface and ice forms. The lower density ice floats, so that lakes freeze from the surface downward.



12.2 Heat Capacity

- When an object at one temperature is placed near or in contact with another object at a higher temperature, energy is transferred to the cooler object, and its temperature rises.

- **Heat capacity is the ratio of the amount of energy transferred to the temperature change.**
- Thermal energy (or heat) transferer occurs because of a temperature difference or by doing work on the substance as, for example, by stirring a liquid or compressing a gas.
- Suppose a small amount of heat ΔQ is transferred to n moles of a substance.

- The **molar heat capacity** C **is defined as the ratio of the heat added per mole to the rise in temperature**

$$C = \frac{1}{n}\frac{\Delta Q}{\Delta T}$$

- Substances such as water, which have a high molar heat capacity, experience relatively small temperature changes when a given amount of heat is transferred.

12.2 Heat Capacity

- The **Specific heat capacity, c is the heat required for a unit temperature change in a unit mass of a substance**. It is related to C by

$$c = \frac{C}{M}$$

- In terms of c, the heat required ΔQ for a temperature change ΔT in a mass m is

$$\Delta Q = mc\,\Delta T$$

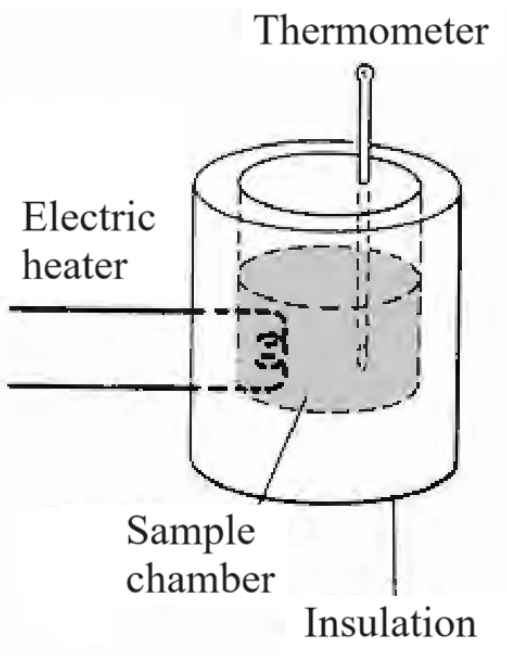
- A simple but effective way of measuring heat capacities at constant pressure uses a calorimeter. Heat ΔQ is supplied by an electrical heater to a well-insulated calorimeter that holds the sample, and a thermometer measures the temperature rise ΔT.
- A sample with mass m and specific heat capacity c absorbs an amount of heat equal to $mc\Delta T$.
- The container also absorbs some heat; if its mass is m_c, and its specific heat capacity is c_c., this heat equals $m_c c_c \Delta T$.
- Adding the heat absorbed by the sample and the container, we have

$$\Delta Q = mc\,\Delta T + m_c c_c\,\Delta T$$

TABLE 12.2

Specific heat capacities at constant pressure at 25° C, except where noted, of various substances in kJ kg^{-1} K^{-1}

Substance	Specific Heat Capacity, c_P
Aluminum	0.898
Steel	0.447
Gold	0.129
Silver	0.234
Diamond	0.518
Lead	0.130
Copper	0.385
Helium (gas)	5.180
Hydrogen (H_2) (gas)	14.250
Iron	0.443
Nitrogen (N_2) (gas)	1.040
Oxygen (O_2) (gas)	0.915
Water (liquid)	4.169
Ice (−10° to 0° C)	2.089
Steam (100° to 200° C)	1.963



12.2 Heat Capacity

Example 12.3

There are 0.1 kg of carbon in a calorimeter at 15 °C. The container has a mass of 0.02 kg and is made of aluminum. The addition of 0.892 kJ of heat energy brings the temperature to 28 °C. What is the specific heat capacity of carbon? Assume the specific heat capacity of aluminum in this temperature range is $0.9\ \text{kJ kg}^{-1}\ \text{K}^{-1}$.

Answer:

We have

$m = 0.1\ \text{kg}$, $m_c = 0.02\ \text{kg}$, $c_c = 0.9\ \text{kJ kg}^{-1}\ \text{K}^{-1}$ and $\Delta T = 28\ °\text{C} - 15\ °\text{C} = 13\ °\text{C}$

$$\Delta Q = mc\,\Delta T + m_c c_c\,\Delta T$$

$$c = \frac{\Delta Q - m_c c_c\,\Delta T}{m\,\Delta T}$$

$$= \frac{0.892\ \text{kJ} - (0.02\ \text{kg})(0.9\ \text{kJ kg}^{-1}\ \text{K}^{-1})(13\ \text{K})}{(0.10\ \text{kg})(13\ \text{K})}$$

$$= 0.506\ \text{kJ kg}^{-1}\ \text{K}^{-1}$$

12.2 Heat Capacity

Example 12.4

A copper pipe of mass 0.5 kg is originally at 20 °C. If its ends are capped after 0.6 kg of water at 98 °C is poured into it, what is the final temperature of the pipe? (Assume the pipe is insulated so no heat is lost to the surroundings.)

Answer:

We have

$m_{pipe} = m_{\mathrm{Cu}} = 0.5\ \mathrm{kg},\ \ \mathrm{c_{pipe}} = \mathrm{c_{Cu}} = 0.358\mathrm{kJ\ kg^{-1}\ K^{-1}}\ \ \ \ \Delta T_1 = T_f - 20\ °\mathrm{C}$

$m_{water} = 0.6\ \mathrm{kg},\ \ c_{water} = 4.169\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$ and $\Delta T = 98\ °\mathrm{C} - T_f = 13\ °\mathrm{C}$

Thermal energy is transferred from the water to the pipe until both are at the same temperature, T_f.

The heat transferred to the pipe is equal to The heat lost by the water

$$\Delta Q_{Cu} = \Delta Q_{water}$$

$$m_{Cu} c_{Cu} \Delta T_{Cu} = m_{water} c_{water} \Delta T_{water}$$

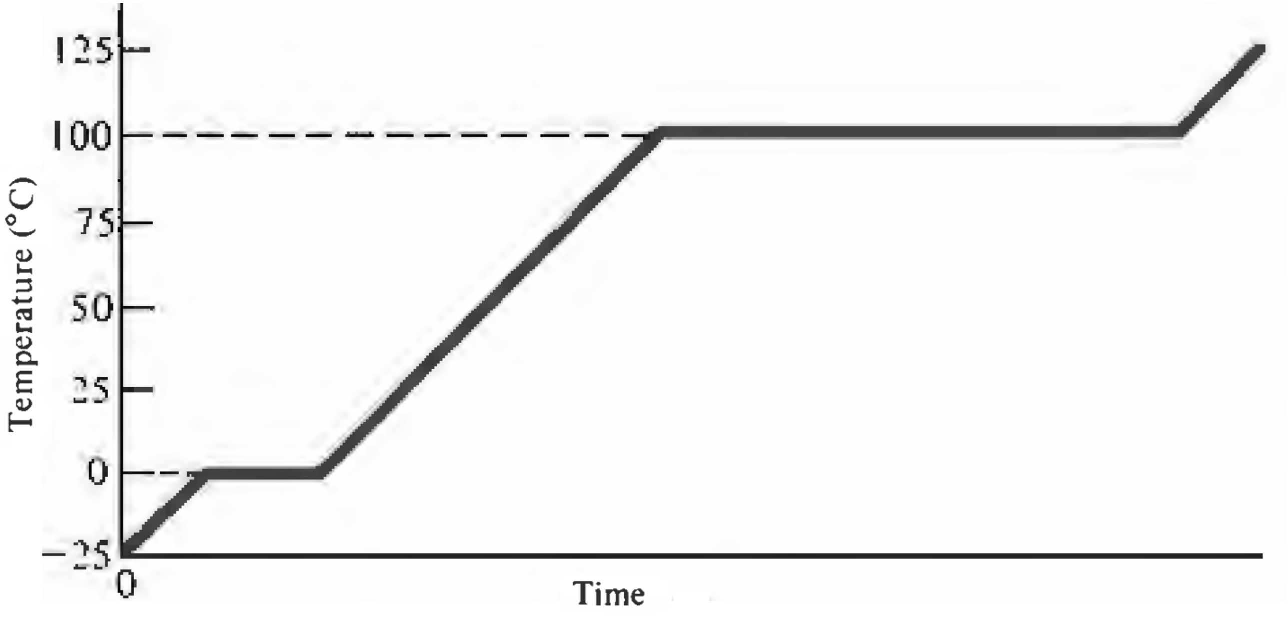
$$(0.5\ \mathrm{kg})(0.385\ \mathrm{kJ\ kg^{-1}\ K^{-1}})(T_f - 20°\ \mathrm{C}) = (0.6\ \mathrm{kg})(4.169\ \mathrm{kJ\ kg^{-1}\ K^{-1}})(98°\ \mathrm{C} - T_f)$$

Multiplying this out and solving for T_f, we find

$T_f = 92.43\ °\mathrm{C}$

12.3 Phase changes

- Most substances can exist in solid, liquid, or gas phases. For example, water may be ice, liquid, or steam.
- **A transition from one of these phases to another is called a phase change.**
- The temperature at which a phase change occurs usually depends on additional variables, such as pressure.



- At temperature below the melting (freezing) temperature, the water can exist only as ice (solid phase)
  - Now as more heat is added, the temperature does not rise. Instead, the ice gradually melts into water, and the temperature remains constant until all the ice is melted.
  - After melting all the ice, as more heat is added, the temperature of the liquid again steadily increases until reaching the boiling temperature.
  - At the boiling temperature, here again the temperature remains constant until all the liquid has been converted into water vapor.
  - Additional heat will now increase the temperature of the gas.

12.3 Phase changes

- **The energy absorbed or liberated in a phase change is called the latent heat.**
- At atmospheric pressure, **the latent heat of fusion L_f** needed to melt ice is $333\ \mathrm{kJ\ kg^{-1}}$.
- **The latent heat of vaporization L_v** needed to boil water at atmospheric pressure is $2255\ \mathrm{kJ\ kg^{-1}}$.
- Some other latent heats are given in the near table.
- **The heat ΔQ needed to change the phase of a mass m is**

$$\Delta Q = Lm$$

TABLE 12.3

Latent heats at atmospheric pressure

Substance	Melting Point (°C)	Latent Heat of Fusion ($\mathrm{kJ\ kg^{-1}}$)	Boiling Point (°C)	Latent Heat of Vaporization ($\mathrm{kJ\ kg^{-1}}$)
Helium			−268.9	21
Nitrogen	−209.9	25.5	−195.8	201
Ethyl alcohol	−114	104	78	854
Mercury	−39	11.8	357	272
Water	0	333	100	2255
Lead	327	24.5	1620	912
Silver	960	88.3	2193	2335
Gold	1063	64.4	2660	1580

12.3 Phase changes

Example 12.5

How much heat is required to melt 5 kg of ice at 0°C?

Answer

Since the latent heat of fusion $L_f = 333\ kJ\ Kg^{-1}$ then

$$Q = Lm$$
$$Q = L_f m = (333\ \text{kJ kg}^{-1})(5\ \text{kg}) = 1665\ \text{kJ}$$

12.3 Phase changes

Example 12.6

If 20 kg of water at 95 °C is mixed with 5 kg of ice at 0 °C, what is the final temperature of the mixture?

Answer

It is useful to determine first whether enough thermal energy is available in the water to melt the ice. If not, the equilibrium temperature will be 0 °C with only a portion of the ice melting. The quantity of heat needed to melt the 5 kg of ice at at 0°C is

$$Q = L_f m = (333 \text{ kJ kg}^{-1})(5 \text{ kg}) = 1665 \text{ kJ}$$

The quantity of heat given by cooling water from 95 °C to 0 °C, it gives up a heat of

$$\Delta Q = mc_P \Delta T = (20 \text{ kg})(4.169 \text{ kJ kg}^{-1} \text{ K}^{-1})(95 \text{ K}) = 7921 \text{ kJ}$$

By comparison, this heat is more than enough to melt the 5 kg of ice at at 0 °C so the final temperature will be above 0 °C.

If the final temperature is T_f, the heat transferred to the ice equal to that given up by the water:

$$L_f\, m_{ice} + m_{ice}(T_f - 0°\text{C}) = m_{water} c_{water}(95°\text{C} - T_f)$$

$$1665\ kJ + (5\ kg)(T_f) = (20\ kg)(4.169)((95\ °\text{C} - T_f)$$

Solving for T_f, we find $T_f = 60.0\ °\text{C}$

12.3 Phase changes

Example 12.7

A 0.6-kg pitcher of tea at 50 ℃ is cooled with 0.4 kg of ice cubes at 0 ℃. What is the equilibrium condition if no heat is lost to the surroundings?

Answer

The heat needed to melt all of the ice is

$$\Delta Q = L_f m_i = (333\ \text{kJ kg}^{-1})(0.4\ \text{kg}) = 133.2\ \text{kJ}.$$

The heat given up by the tea if it is cooled to 0 ℃ is

$$\Delta Q = (0.6\ \text{kg})(4.169\ \text{kJ kg}^{-1}\ \text{K}^{-1})(50\ \text{K}) = 125.1\ \text{kJ}$$

There is no enough heat to melt all the ice. Thus, some of the ice will not melt and the final temperature will be 0 ℃.

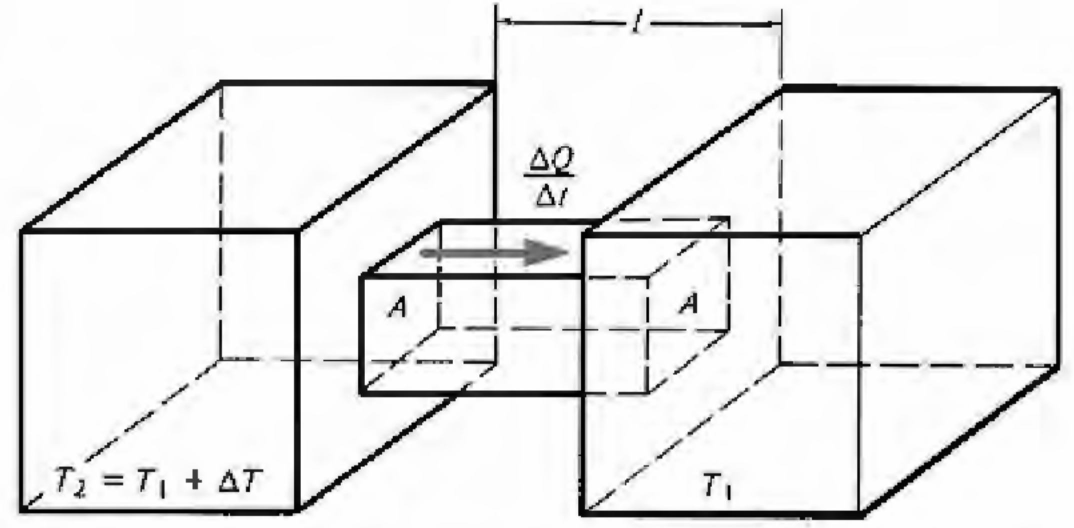
To find the mass of the melted ice, m_1, we equate the heat loss of the cooled tea to the heat needed to melt a mass m_1 of ice

$$\Delta Q = m_1 L_f$$

$$m_1 = \frac{\Delta Q}{L_f} = \frac{125.1\text{kJ}}{333\ \text{kJ kg}^{-1}} = 0.376\ \text{kg}$$

12.4 Heat conduction

- Heat transfer always occurs from regions of higher temperature to regions of lower temperature, so that two objects isolated from their surroundings gradually approach a common temperature.
- Heat transfer may be by conduction, convection and radiation.
- When two objects at temperatures T_1 and T_2 are connected by a rod, their temperature difference $\Delta T = T_2 - T_1$ will steadily diminish.
- The rate at which heat flows from the hotter to the colder object is proportional to
  - the cross-sectional area A.
  - the ratio $\frac{\Delta T}{l}$, which is called the temperature gradient.
- Thus, the heat flow is



$$H = \frac{\Delta Q}{\Delta t} = \kappa A \frac{\Delta T}{l}$$

Here κ is a proportionality constant called the thermal conductivity.

12.4 Heat conduction

- One of the most important insulators is air. Insulation in homes and in the material for warm clothing utilize this fact. The fibers of the material trap air in the material, and this air acts as an insulator.

TABLE 12.4

Thermal conductivities in $W\ m^{-1}\ K^{-1}$

Substance	Thermal Conductivity, κ
Silver	420
Copper	400
Aluminum	240
Steel	79
Ice	1.7
Glass, concrete	0.8
Water	0.59
Animal muscle, fat	0.2
Wood, asbestos, fiberglass	0.08
Felt, rock wool	0.04
Air	0.024
Down	0.019
Styrofoam	0.01

- Body tissue is also a good insulator. When the environment is warm, the interior body temperature is quite uniform (Figure a). Because body tissues are poor conductors, the inner core of the body can be kept warm in a cold environment (Figure b).