

# Chapter 1

## Motion in a straight line

**Dr. Ghassan Alna'washi**

# COURSE TOPICS:

- 1.1 Measurements, Standards and Units
- 1.2 Displacements; Average Velocity
- 1.3 Instantaneous Velocity
- 1.4 Acceleration
- 1.5 Finding the Motion of an Object
- 1.6 The Acceleration of Gravity and Falling Objects

Examples to be explained and solved:

1.2; 1.4; 1.14; 1.16 and 1.20



# 1.1 Measurements, standards, units and errors

Physical quantities are classified into **fundamental quantities** such as **mass, length, time** and **derived quantities** such as **velocity, acceleration, force, energy...**

**Primary standard:**

- **Length** has a dimension **L**
- **Time** has a dimension **T**
- **Mass** has a dimension **M**

All mechanical quantities can be expressed in terms of some combination of these three fundamental dimensions.

**Example:**

$$velocity = v = \frac{distance}{time}$$

$$[v] = \frac{[distance]}{[time]} = \frac{L}{T}$$

# 1.1 Measurements, standards, units and errors

## Systems of units

International system (S.I)			British system		C.G.S System	
Physical quantity	unit	symbol	unit	symbol	unit	symbol
Length	Meter	$m$	Foot	$ft$	centimeter	$cm$
			$1ft = 0.3048 m$		$1cm = 10^{-2}m$	
Mass	kilogram	$kg$	Pound	$lb$	gram	$g$
			$1lb = 0.4536 kg$		$1g = 0.001 kg$	
Time	second	$s$	Second	$s$	second	$s$

- The international system is also known as the *metric system* or the *m.k.s system*.
- In the medical area some units are more used than those of the S.I units, such as the use of calorie as unit for energy than the Joule ( $1 cal = 4.2 J$ ), the millimeter of mercury for the pressure than the Pascal ( $1mmHg = 133.32 Pa$ ) or the liter for the volume than the meter cube.

# 1.1 Measurements, standards, units and errors

## The scientific notation

- A number is said to be in scientific notation when it is written as a number between 1 and 10,
- times a power of 10.
- For example, 521 can be written as  $5.21 \times 10^2$ , or a small number like 0.000000521 can be written as  $5.21 \times 10^{-7}$ .
- The advantage of this notation is its compactness, it also facilitates numerical calculations.
- When a number is written with the powers of 10, we can use the following **prefixes**

Multiples		Prefix	symbol	Sub-multiples		Prefix	symbol
10		deca	da	0.1	$10^{-1}$	deci	d
100	$10^2$	hecto	h	0.01	$10^{-2}$	centi	c
1000	$10^3$	kilo	k	0.001	$10^{-3}$	milli	m
1000 000	$10^6$	Mega	M	0.000001	$10^{-6}$	micro	$\mu$
1000 000 000	$10^9$	Giga	G	0.000 000 001	$10^{-9}$	nano	n
1000 000 000 000	$10^{12}$	Tera	T	0.000 000 000 001	$10^{-12}$	pico	p

# 1.1 Measurements, standards, units and errors

## Conversion of units

To convert quantities from a unit system to another, we can use the following **systematic method**:

Suppose we want to convert a length  $L = 1.75 \text{ m}$  into foot . The conversion factor between the two units is given by:  $1 \text{ ft} = 0.3048 \text{ m}$

To convert from meter to foot we have to follow these steps:

**1. Multiply the quantity to convert by one:**

$$L = 1.75 \text{ m} \times 1$$

**2. Rearrange the conversion factor in quotient equal to 1 that allows the elimination of the unit from which we want to convert :**

$$1 \text{ ft} = 0.3048 \text{ m} \Rightarrow \frac{1 \text{ ft}}{0.3048 \text{ m}} = 1$$

**3- Replace this form in the first step:**

$$L = 1.75 \text{ m} \times (1) = 1.75 \text{ m} \times \frac{1 \text{ ft}}{0.3048 \text{ m}} = 5.74 \text{ ft}$$

# 1.1 Measurements, standards, units and errors

## Conversion of units

**Example 1.1:** Convert 100 *ft* into meters.

$$100 \text{ ft} = 100 \text{ ft} \times (1) = 100 \text{ ft} \times \left(\frac{0.3048\text{m}}{1\text{ft}}\right) = 30.48 \text{ m}$$

## **Example 1.2:**

Convert the velocity of 24 *m/s* into *km/h*.

We have 1 *km* = 10<sup>3</sup> *m* and 1 *h* = 3600 *s*

Then 24 *m/s* = 24  $\frac{\text{m}}{\text{s}}$  × (1) × (1) { (1) *is written twice because we have two units to convert* }

$$24 \text{ m/s} = 24 \frac{\cancel{\text{m}}}{\text{s}} \times \left(\frac{1\text{km}}{10^3\cancel{\text{m}}}\right) \times \left(\frac{3600\cancel{\text{s}}}{1\text{h}}\right) = \frac{24 \times 3600}{10^3} \frac{\text{km}}{\text{h}} = 86.4 \text{ km/h}$$

# 1.1. Measurements, standards, units and errors

## Conversion of units

**Example 1.3:** The skin is the largest organ in the human body; for a human adult, the average area of the skin surface is about  $1.8 \text{ m}^2$ , how much squared foot is this area?

Solution: Given that  $1 \text{ ft} = 0.3048 \text{ m}$ , then  $1 \text{ ft}^2 = (0.3048)^2 \text{ m}^2$

$$A = 1.8 \text{ m}^2 = 1.8 \cancel{\text{m}^2} \times \left( \frac{1 \text{ ft}^2}{(0.3048)^2 \cancel{\text{m}^2}} \right) = 19 \text{ ft}^2$$

**Example 1.4:** an ampoule contains a solution of drug of  $300 \mu\text{g}/5 \text{ ml}$ , convert this dose into  $\text{g}/\text{l}$ .

Solution:  $\frac{300 \mu\text{g}}{5 \text{ ml}} = \frac{300 \times 10^{-6} \text{ g}}{5 \times 10^{-3} \text{ l}} = 0.06 \text{ g/l}$



# 1.1 Measurements, standards, units and errors

## Conversion of units

**Example 1.4:** Convert a velocity of  $60 \text{ mi h}^{-1}$  (mile per hour) to  $\text{m s}^{-1}$  (meter per second)

Multiplying  $60 \text{ mi h}^{-1}$  by 1 twice gives

$$\begin{aligned}(60 \text{ mi h}^{-1})(1)(1) &= (60 \text{ mi h}^{-1})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(1609 \text{ m mi}^{-1}) \\ &= 60 \left(\frac{1609}{3600}\right) \text{ m s}^{-1} = 26.8 \text{ m s}^{-1}\end{aligned}$$

# 1.1 Measurements, standards, units and errors

## Types of Errors

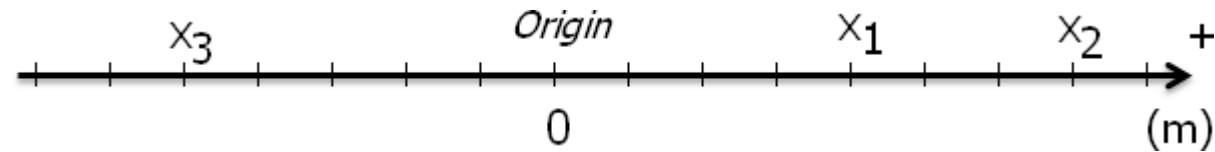
Measurements and predictions are both subject to errors.

Measurement errors are of two types:

- **Random errors:** affects precision (how reproducible the same measurement is under equivalent circumstances).
- **Systematic errors:** affects the accuracy of a measurement (how close the observed value is to be true)

## 1.2 Displacement; Average velocity

- To describe the motion of an object we should first set up a **coordinate system** to locate the position.
- A **coordinate system** is made up of an origin, a positive direction and a unit of length.
- **Position (x)**: is the location of an object with respect to a chosen reference.
  - It could be positive or negative
  - S.I unit is meter



**Example:** In the above figure:

$$x_1 = 4 \text{ m}, x_2 = 7 \text{ m}, x_3 = -5 \text{ m}$$

- **Displacement:** is the change in the position:

$$\Delta x = x_{\text{final}} - x_{\text{initial}} = x_f - x_i$$

- The displacement can be positive or negative; it is negative if the motion is in the negative direction
- It is not the same as the distance travelled

**Example:** In the figure above, if the object moves from  $x_1$  to  $x_2$ :  $\Delta x = x_2 - x_1 = 7 \text{ m} - 4 \text{ m} = 3 \text{ m}$ .

The displacement from  $x_2$  to  $x_3$  is:  $\Delta x = x_3 - x_2 = -5 \text{ m} - 7 \text{ m} = -12 \text{ m}$ .

## 1.2 Displacement; Average velocity

- **Average velocity:** the average velocity is the displacement over an elapsed time  $\Delta t$ :

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} \Rightarrow \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- It could be positive or negative
- S.I unit is m/s
- Direction of the average velocity is the same as that of the displacement

**Example:** A car is at  $x_1 = 600 \text{ m}$  when  $t_1 = 5 \text{ s}$  and at  $x_2 = 500 \text{ m}$  when  $t_2 = 15 \text{ s}$ .  
What is the average velocity?

**Solution:**

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{(500 - 600) \text{ m}}{(15 - 5) \text{ s}} = -10 \text{ m/s}$$

## 1.2 Displacement; Average velocity

**Example:** The position of a car in successive time intervals is represented in the following figure.

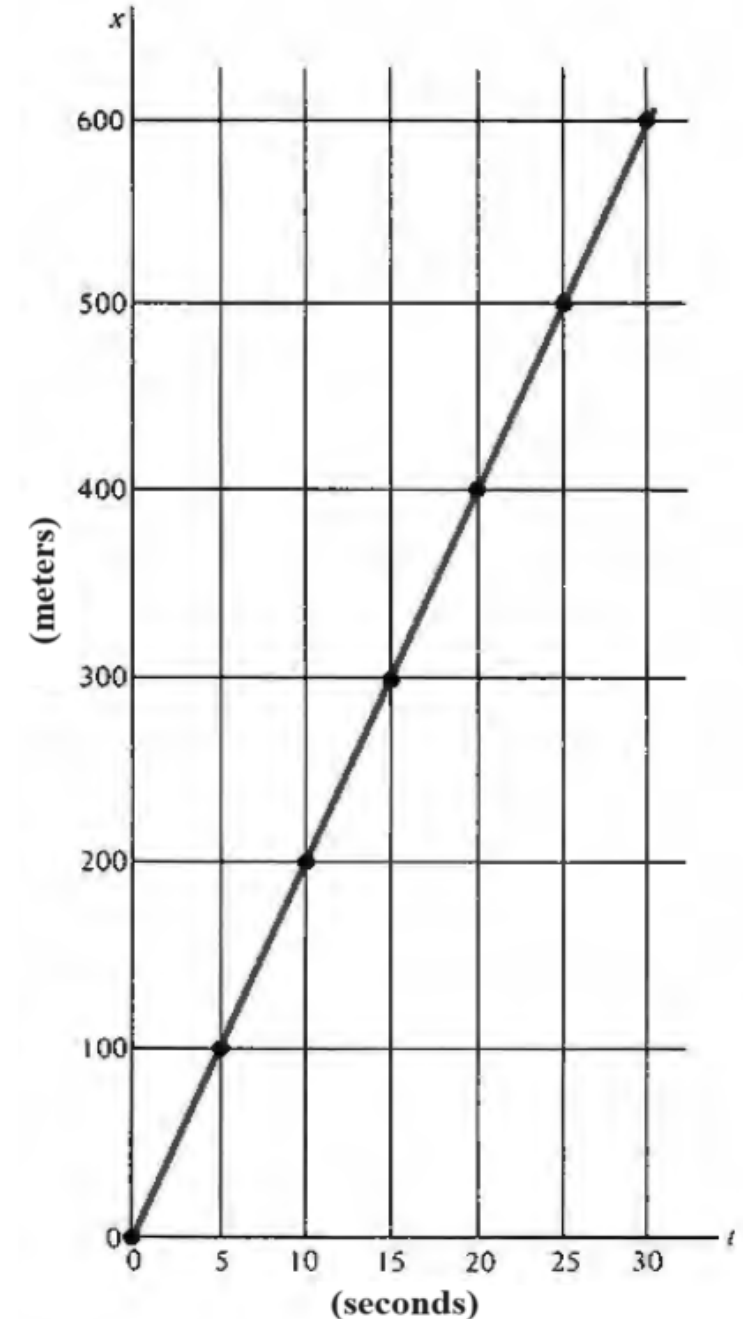
a. Find the average velocity in the time interval 0 to 10 s

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{(200 - 0) \text{ m}}{(10 - 0) \text{ s}} = 20 \text{ m/s}$$

b. Find the average velocity in the time interval 5 s to 20 s

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{(400 - 100) \text{ m}}{(20 - 5) \text{ s}} = 20 \text{ m/s}$$

- Because the car in this example moves equal distances in equal times, the average velocity will be the same no matter what time interval is chosen. In this situation the motion is said to be **uniform**.
- Motion that is not uniform is said to be **accelerated**.



## 1.2 Displacement; Average velocity

**Example:** A car moves as shown in figure. Find its average velocity from  $t = 0$  to  $t = 1$  s and from  $t = 1$  s to  $t = 2$  s .

**Solution:**

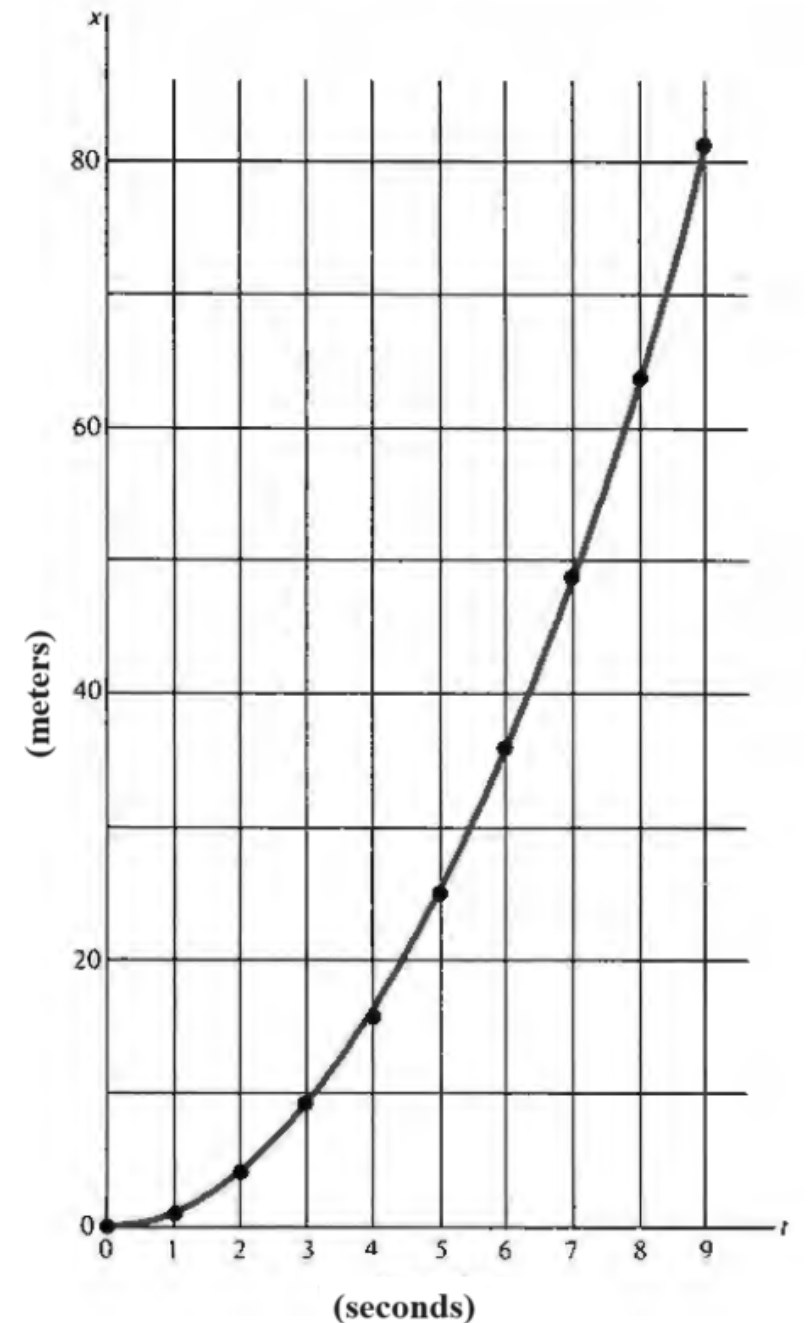
from  $t = 0$  to  $t = 1$  s, the average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{(1-0) \text{ m}}{(1-0) \text{ s}} = 1 \text{ m/s}$$

from  $t = 1$  s to  $t = 2$  s, the average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{(4-1) \text{ m}}{(2-1) \text{ s}} = 3 \text{ m/s}$$

Note that the average velocity is not constant because the car is accelerated.



## 1.2 Displacement; Average velocity

**Example:** A particle moved from  $x_1 = 2 \text{ m}$  to  $x_2 = -5 \text{ m}$  in time of 2 seconds and then back to  $x_3 = x_1 = 2 \text{ m}$  in time of 2 seconds. Find

- The displacement from  $x_1$  to  $x_2$ .
- The average velocity between  $x_1$  and  $x_2$ .
- The displacement when the particle returns to its initial position  $x_1$ .
- The average velocity in the whole trip

### Solution

$$a. \quad \Delta x = x_2 - x_1 = -7 \text{ m}$$

$$b. \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{-7 \text{ m}}{2 \text{ s}} = -3.5 \text{ m/s}$$

$$c. \quad \Delta x = x_3 - x_1 = 0 \text{ m}$$

$$d. \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{0 \text{ m}}{4 \text{ s}} = 0$$

Note that the displacement in the example is zero but the distance travelled is

$$d = 7 \text{ m} + 7 \text{ m} = 14 \text{ m}$$

## 1.2 Displacement; Average velocity

**Example:** A car moves along a straight highway at an average velocity of 100 km/h.

(a) How far will it go in 2 h?

(b) How long will it take to travel 350 km?

**Solution**

$$a. \quad \bar{v} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = \bar{v} \Delta t = \left(100 \frac{\text{km}}{\text{h}}\right) (2 \text{ h}) = 200 \text{ km} = 2 \times 10^5 \text{ m}$$

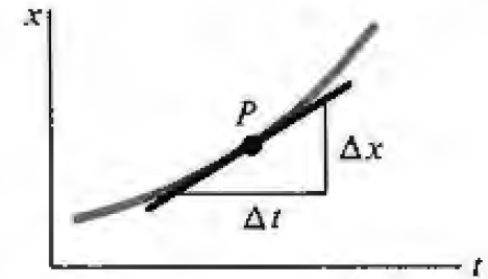
$$b. \quad \Delta t = \frac{\Delta x}{\bar{v}} = \frac{350 \text{ km}}{100 \text{ km/h}} = 3.5 \text{ h} = 3.5 \text{ h} \times \frac{3600 \text{ s}}{\text{h}} = 12600 \text{ s}$$



## 1.3 Instantaneous velocity

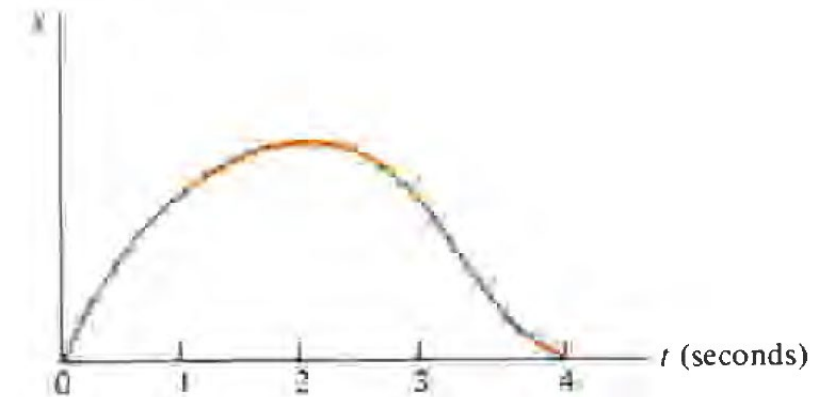
- The average velocity doesn't give a description about the rate of change of the position at each instant. We need often to know the velocity of an object at each second, referred to as **instantaneous velocity**:
- **Instantaneous velocity**: The instantaneous velocity is determined by computing the average velocity for an extremely short time interval:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \equiv \text{slope of the tangent of } x - t \text{ graph at a definite time } t$$



The position versus time of an object is represented in the lowest graph:

- Between  $t = 0$  and  $t = 2$  s, the  $x$ - $t$  curve is rising. The slope and  $v$  are positive
- At  $t = 2$  s,  $x$  has its greatest value. The curve is flat there; Hence the slope and  $v$  are zero.
- After  $t = 2$  s, the  $x$  is decreasing, so  $v$  is negative.



## 1.3 Instantaneous velocity

### Example:

the motion of an object is given by the equation :  $x(t) = 2 + 3t - 2t^2$  , where  $x$  is the position in meter and  $t$  is the instant in second.

- (a) Find the velocity of the object at  $t = 5$  s.  
(b) What is its average velocity between the two instants  $t_1 = 3$ s and  $t_2 = 5$ s?

### **Answer:**

(a)  $v = \frac{dx}{dt} = 3 - 4t$ ; then at the instant  $t = 5$  s,  $v = 3 - 4 \times 5 = -17$  m/s

(b) At  $t_1 = 3$ s :  $x_1 = 2 + 3 \times 3 - 2 \times 3^2 = -7$  m

At  $t_2 = 5$ s :  $x_2 = 2 + 3 \times 5 - 2 \times 5^2 = -33$ m

Then the average velocity is:

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\{-33 - (-7)\}}{5 - 3} = -13 \text{ m/s}$$

# 1.4 Acceleration

- Like position, velocity can change with time. The rate at which velocity changes is the *acceleration*
- Again, we can discuss the average and the instantaneous acceleration.
- **The Average acceleration** is the change of velocity over an interval of time  $\Delta t$ :

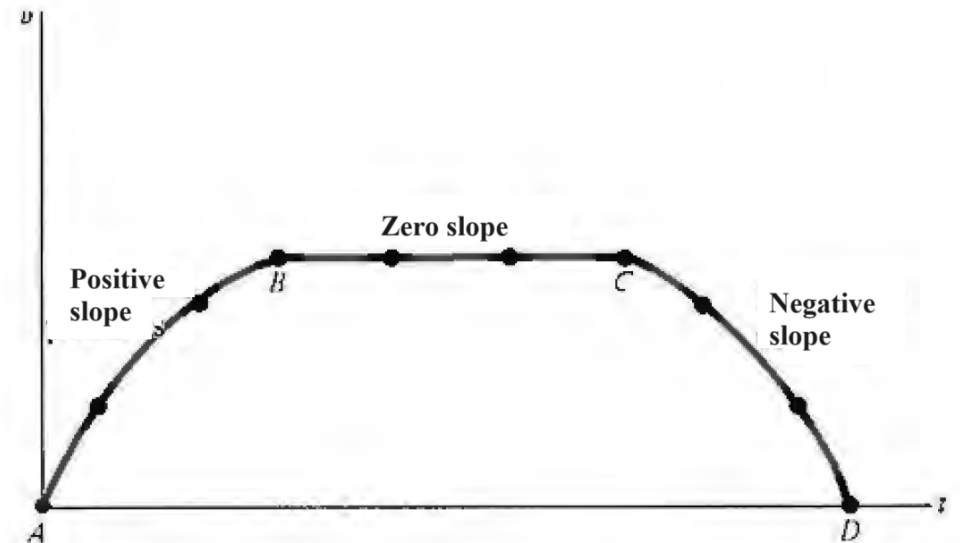
$$\bar{a} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- S.I unit is  $m/s^2$
  - The acceleration could be positive or negative
- **The Instantaneous acceleration** is the rate change of velocity over an extremely short time interval

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \equiv \text{slope of the tangent of } v-t \text{ graph at a definite time } t$$

The graph represent velocity-versus-time for a car.

The slope and acceleration are positive from A to B, zero from B to C, and negative from C to D.



## 1.4 Acceleration

**Example:** the motion of an object is given by the equation :  $x(t) = 27t - 4t^2$ .  $x$  is *in meter* and  $t$  *in second*. Find the velocity and acceleration of the object at  $t = 5$  s.

Answer:

$$v(t) = \frac{dx}{dt} = 27 - 8t \Rightarrow v(t = 5 \text{ s}) = 27 - 8 \times 5 = -13 \text{ m/s}$$

$$a = \frac{dv}{dt} = -8 \text{ m/s}^2$$

the motion is with constant acceleration

## 1.4 Acceleration

**Example:** the motion of an object is given by the equation :  $x(t) = 5t^2 - 2t + 4$ .  $x$  is in meter and  $t$  in second. Find

- the position of the object at  $t = 0$  s and at  $t = 2$  s.
- The average velocity in the time interval  $[0, 2]$  s.
- The instantaneous velocity at  $t = 0$  s and at  $t = 2$ .
- The average acceleration in the time interval from  $t = 0$  to  $t = 2$  s.
- The instantaneous acceleration

**Answer:**

$$a. \quad x(0) = 5 \times 0^2 - 2 \times 0 + 4 = 4 \text{ m}, \quad x(2) = 5 \times 2^2 - 2 \times 2 + 4 = 20 \text{ m}$$

$$b. \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(2) - x(0)}{(2 - 0)} = \frac{(20 - 4) \text{ m}}{2 \text{ s}} = 8 \text{ m/s}$$

$$c. \quad v(t) = \frac{dx}{dt} = 10t - 2 \Rightarrow \\ v(t = 0) = 10 \times 0 - 2 = -2 \text{ m/s} \quad \text{and} \quad v(t = 2 \text{ s}) = 10 \times 2 - 2 = 18 \text{ m/s}$$

$$d. \quad \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v(t = 2 \text{ s}) - v(t = 0)}{2 - 0} = \frac{\{18 - (-2)\} \text{ m/s}}{2 \text{ s}} = 10 \text{ m/s}^2$$

$$e. \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d}{dt} (10t - 2) = 10 \text{ m/s}^2 \Rightarrow \quad \text{the motion is with constant acceleration}$$

# 1.4 Acceleration

**Example:** the velocity of a car is given by the equation :  $v = 20 - 3t$ .  $v$  is in m/s and  $t$  in second. Find

- a. The average acceleration from  $t = 1$  s and at  $t = 3$  s.
- c. The instantaneous acceleration.
- d. The position at  $t = 1$  s if you are given that the car starts its motion at the origin.

**Answer:**

$$a. \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v(t=3 \text{ s}) - v(t=1 \text{ s})}{(3-1)\text{s}} = \frac{\{(20-3 \times 3) - (20-3 \times 1)\} \text{ m/s}}{2 \text{ s}} = -3 \text{ m/s}^2$$

$$b. a = \frac{dv}{dt} = \frac{d}{dt}(20 - 3t) = -3 \text{ m/s}^2$$

$$c. v = \frac{dx}{dt} \implies dx = v dt$$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v dt \implies x(t_f) - x(t_i) = \int_{t_i}^{t_f} v dt$$

$$x(t = 1 \text{ s}) - x(t = 0) = \int_0^1 (20 - 3t) dt = (20t - \frac{3}{2}t^2) \Big|_0^1 = 18.5 \text{ m}$$

$$x(t = 1 \text{ s}) - 0 = 18.5 \text{ m}$$

$$x(t = 1 \text{ s}) = 18.5 \text{ m}$$

## 1.5 Finding the motion of an object

- If the initial position ( $x_i$ ) and velocity ( $v_i$ ) are known, then, at later time  $t$ , the final position ( $x_f$ ) and velocity ( $v_f$ ) can then be found if the acceleration is given.
- When the acceleration is constant, we can find the equations of motion. In this case the average and the instantaneous accelerations are equal. and the following equations of motion with constant acceleration are obtained:

Average velocity	$\bar{v} = \frac{1}{2}(v_f + v_i) \dots \dots (1)$	Relating the final velocity to the initial velocity
Velocity equation	$v_f = v_i + a\Delta t \dots \dots (2)$	Relating the final velocity to the initial velocity and the acceleration
Equation of motion	$\Delta x = x_f - x_i = v_i \Delta t + \frac{1}{2}a(\Delta t)^2 \dots \dots (3)$	Relating the final position to the initial position, the initial velocity and the acceleration
	$v_f^2 = v_i^2 + 2 a \Delta x$ $= v_i^2 + 2a(x_f - x_i) \dots \dots (4)$	Relating the final velocity to the initial velocity, the acceleration and the position change

## 1.5 Finding the motion of an object

### Example 1.16 P 14:

A car, initially at rest at a traffic light, accelerates at  $2 \text{ m/s}^2$  when the light turns green. After 4 seconds what are its velocity and position?

### *Solution:*

Since we know the acceleration  $a$ , the elapsed time  $\Delta t$ , and the initial velocity  $v_i = 0$ , we can use Equations (2) and (3) to find the velocity and the displacement. Thus

$$v_f = v_i + a\Delta t = 0 + (2 \text{ m/s}^2)(4 \text{ s}) = 8 \text{ m/s}$$

$$\Delta x = x_f - x_i = v_i t + \frac{1}{2} a(\Delta t)^2$$

$$x_f - 0 = 0 + \frac{1}{2} (2 \text{ m/s}^2)(4 \text{ s})^2 = 16 \text{ m}$$

$$x_f = 16 \text{ m}$$

*After 4s the car has reached a velocity of  $8\text{m}\cdot\text{s}^{-1}$  and is 16m far from the light. Note that we could also find  $\Delta x$  from Eq. (4) using our result for  $v$*

**Exercise:** Resolve example 1.16 with an initial velocity  $v_0 = 3 \text{ m/s}$ .



# 1.5 Finding the motion of an object

## Example

A car accelerates from rest with constant acceleration of  $2 \text{ m/s}^2$  onto a highway where cars are moving steady at  $24 \text{ m/s}$

(a) How long does it take for the car to reach the highway speed.

(b) How far will it travel in that time.

**Answer:**

(a) We have  $v_i = 0$ ,  $v_f = 24 \text{ m/s}$ ,  $a = 2 \text{ m/s}^2$

$$v_f = v_i + a\Delta t \Rightarrow \Delta t = \frac{v_f - v_i}{a} = \frac{(24 - 0) \text{ m/s}}{2 \text{ m/s}^2} = 12 \text{ s}$$

(b)  $\Delta x = x_f - x_i = v_i\Delta t + \frac{1}{2}a(\Delta t)^2 = 0 + \frac{1}{2}(2 \text{ m/s}^2)(12 \text{ s})^2 = 144 \text{ m}$

## 1.6 The acceleration of gravity and Falling objects

*Falling objects* undergo an acceleration, which we attribute to **gravity**, the gravitational attraction of the earth.

If gravity is the only factor affecting the motion of an object falling near the earth's surface, and air resistance is either absent or negligibly small. So long as the object's distance from the surface of the earth is small compared to the earth's radius, it is found that:

- 1. The gravitational acceleration is the same for all falling objects, no matter what their size or composition or mass.*
- 2. The gravitational acceleration is constant. It does not change as the object falls.*

- An object initially thrown upward has also the gravity acceleration. Its speed steadily decreases in magnitude until it becomes zero at the highest point reached.
- Free falling problems can be solved using the equations of motion in a straight line (which is on the vertical direction) with a constant acceleration equal to  $9.80 \text{ m/s}^2$ .
- In the equations of motion, we will use

$$a_y = -g = 9.80 \text{ m/s}^2$$

## 1.6 The acceleration of gravity and Falling objects

In the absence of any other forces, the equation of motion for a freely falling object near the surface of the earth are

$$g = 9.8 \text{ m/s}^2$$

$$a = -g = -9.80 \text{ m/s}^2$$

$$v_f = v_i - g \Delta t$$

$$y_f - y_i = v_{y_i} \Delta t - \frac{1}{2} g (\Delta t)^2$$

$$v_f^2 = v_i^2 - 2g(y_f - y_i)$$

# 1.6 The acceleration of gravity and Falling objects

**Example 1.20 P 17:** A ball is dropped from a window 84 m above the ground.

- (a) when does the ball strike the ground?
- (b) what is its velocity and its speed when it strikes the ground?

**Answer:**

(a) It is given that  $v_i = 0$

$$y_f - y_i = v_{y_i}t - \frac{1}{2}g(\Delta t)^2$$

$$0 - 84 \text{ m} = 0 - \frac{1}{2}(9.8 \text{ m/s}^2) (\Delta t)^2$$

$$\Delta t = \sqrt{\frac{84}{4.9}} \text{ s} = 4.14 \text{ s}$$

$$(b) v_f = v_i - g\Delta t = 0 - (9.8 \text{ m/s}^2)(4.14 \text{ s}) = -40.6 \text{ m/s}$$

then the ball hits the ground with a speed of 40.6 m/s downward

## 1.6 The acceleration of gravity and Falling objects

**Example:** A ball is thrown upward at 19.6 m/s from a window 58.8 m above the ground.

(a) How high does it go?

(b) When does it reach its highest point?

(c) When does it strike the ground?

**Answer**

(a) It is given that  $v_i = 19.6 \text{ m/s}$ ,  $a = -9.8 \text{ m/s}^2$ . At the highest point,  $v_f = 0$

$$v_f^2 = v_i^2 - 2g\Delta x \implies 0 = (19.6 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)\Delta x$$

$$\Delta x = \frac{(19.6 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 19.6 \text{ m}$$

(b)  $v_f = v_i - g\Delta t$

$$0 = (19.6 \text{ m/s}) - (9.8 \text{ m/s}^2)\Delta t$$

$$\Delta t = \frac{19.6 \text{ m/s}}{9.8 \text{ m/s}^2} = 2 \text{ s}$$

(c)  $\Delta x = v_i\Delta t - \frac{1}{2}g(\Delta t)^2 \implies -58.8 = 19.6\Delta t - 4.9(\Delta t)^2 \implies 4.9(\Delta t)^2 - 19.6(\Delta t) - 58.8$

$$\Delta t = \frac{19.6 \pm \sqrt{19.6^2 - 4(4.9)(-58.8)}}{2(4.9)}$$

$\Delta t = 6 \text{ s}$ . Not that the other solution  $\Delta t = -2 \text{ s}$  is rejected.

## Problems:

**1-13** A car travels 30 km in 45 min on a straight highway. What is its average velocity in kilometres per hour ( $\text{km h}^{-1}$ )?

**Answer:**

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{30 \text{ km}}{45 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}}} = 40 \text{ km/h}$$

**1-45** A train traveling with a velocity of  $30 \text{ m s}^{-1}$  stops with a uniform acceleration in 50 s. (a) What is the acceleration of the train? (b) What is the distance traveled before coming to rest?

**Answer:**

(a) We are given:  $v_i = 30 \text{ m/s}$ ,  $v_f = 0$ ,  $\Delta t = 50 \text{ s}$

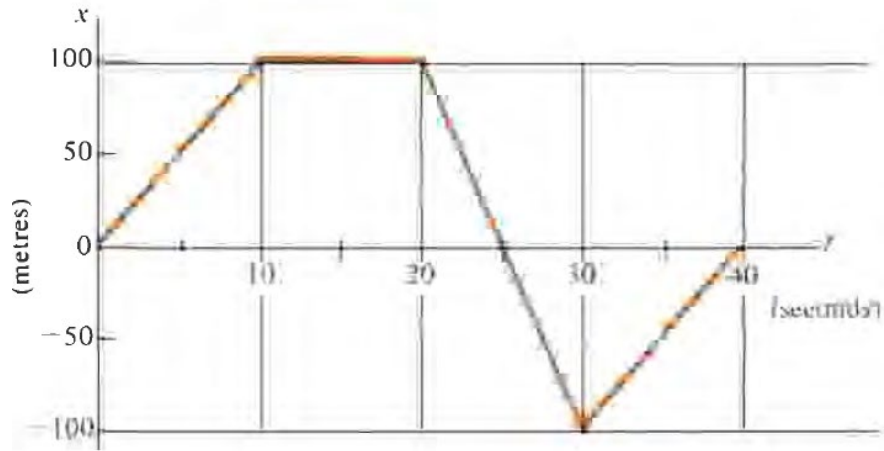
$$v_f = v_i + a\Delta t$$

$$a = \frac{(v_f - v_i)}{\Delta t} = \frac{(0 - 30) \text{ m/s}}{50 \text{ s}} = -0.6 \text{ m/s}^2$$

$$\begin{aligned} \text{(b) } \Delta x &= v_i \Delta t + \frac{1}{2} a (\Delta t)^2 \\ &= (30 \text{ m/s})(50 \text{ s}) + \frac{1}{2} (-0.6 \text{ m/s}^2)(50 \text{ s})^2 \\ &= 750 \text{ m} \end{aligned}$$

## Problems:

**1-28** In Fig. 1.13, what is the instantaneous velocity at (a)  $t = 5$  s; (b)  $t = 15$  s; (c)  $t = 25$  s; (d)  $t = 35$  s?



Answer:

$$v = \frac{dx}{dt} = \text{slope of } x - t \text{ graph}$$

$$(a) \text{ At } t=5 \text{ s: } v = \text{slope} = \frac{(100-0)\text{m}}{(10-0)\text{s}} = 10 \text{ m/s}$$

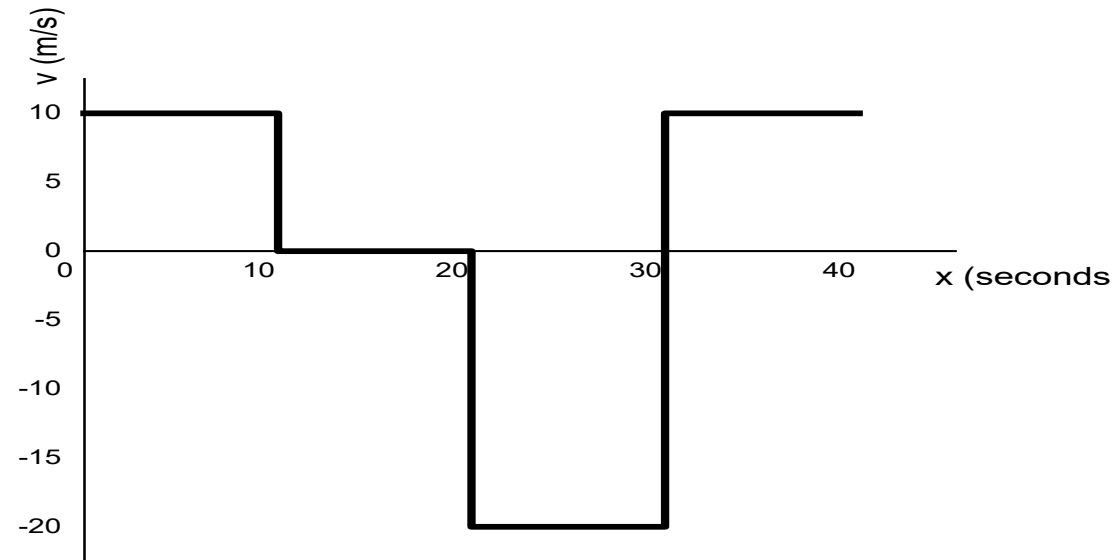
$$(b) \text{ At } t = 15 \text{ s: } v = \text{slope} = 0$$

$$(c) \text{ At } t = 25 \text{ s: } v = \text{slope} = \frac{(-100-100)\text{m}}{(30-20)\text{s}} = -20 \text{ m/s}$$

$$(d) \text{ At } t = 35 \text{ s: } v = \text{slope} = \frac{(0-(-100))\text{m}}{(40-30)\text{s}} = 10 \text{ m/s}$$

**1-29** Draw the instantaneous velocity-versus-time graph corresponding to Fig. 1.13.

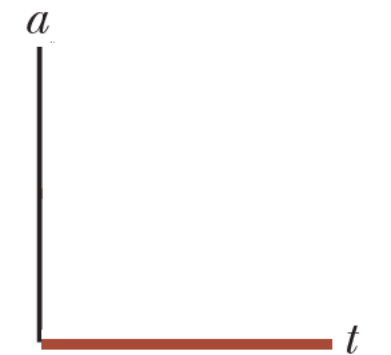
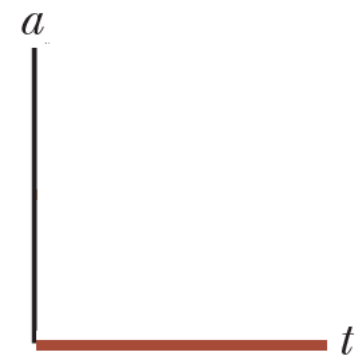
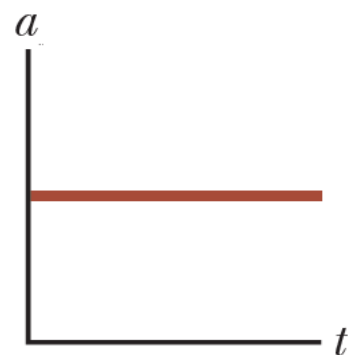
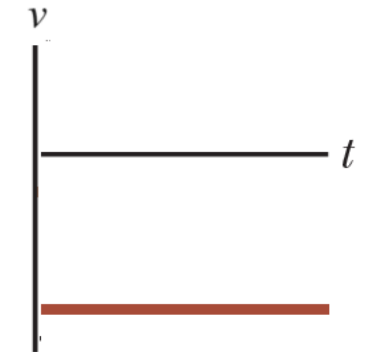
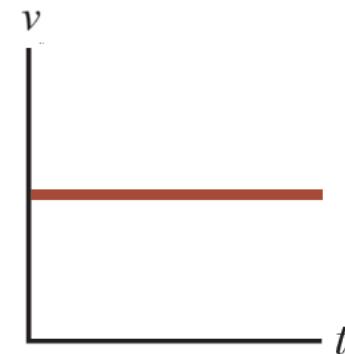
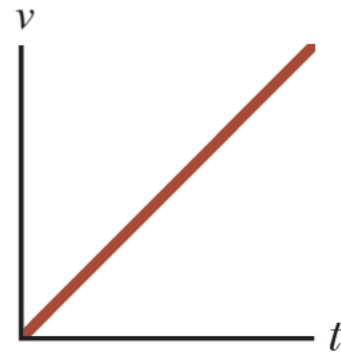
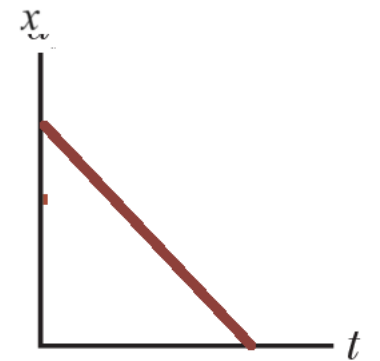
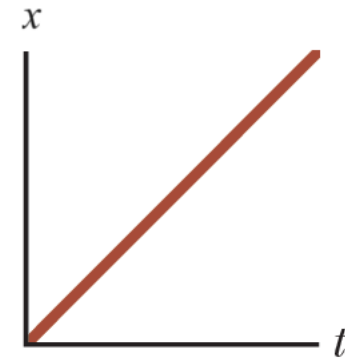
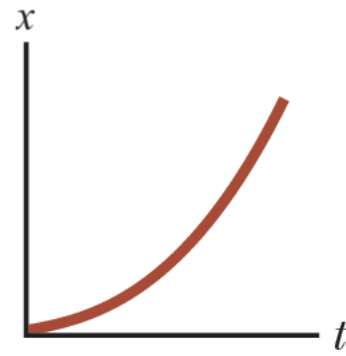
Using the answer of problem 1-28, the v-t graph becomes



# Problems:

Problem:

Draw the acceleration versus time for the following x-t graphs





## Problems:

Problems: (Chapter One): Motion in straight line.

① An object moves along the  $x$ -axis according to the equation:

$$x(t) = 3t^2 - 2t + 3 \text{ find:}$$

- ~~the~~ the average speed between  $t = 2\text{ s}$  to  $t = 3\text{ s}$
- the instantaneous velocity at  $t = 2\text{ s}$  and at  $t = 3\text{ s}$
- the instantaneous acceleration at  $t = 2\text{ s}$  and at  $t = 3\text{ s}$

② a truck moved  $40\text{ m}$  in  $8.5\text{ s}$  by smoothly slowing down to final speed of  $2.8\text{ m/s}$ ; find the initial speed and the acceleration

③ a body moving with uniform acceleration has a velocity of  $12\text{ m/s}$  in the positive  $x$ -direction when its coordinate is  $x = 3\text{ m}$ ; if the coordinate 2 seconds later is  $-5\text{ m}$  find the body's acceleration.

## Problems:

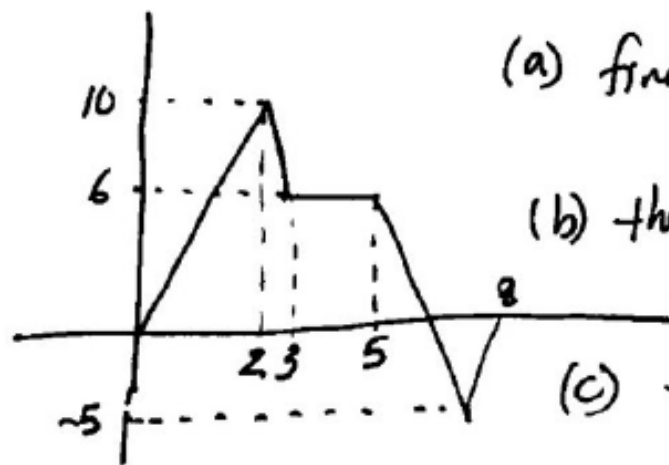
④ a ball is thrown upward from the ground with initial velocity of  $15 \text{ m/s}$

a) how long does it take to reach the maximum height

b) what is the maximum height

c) determine the velocity and the acceleration of the ball at  $t = 2 \text{ s}$

⑤ given the  $x-t$  graph



(a) find the average velocity in the intervals  $[0, 3] \text{ s}$  ;  $[5, 8 \text{ s}]$

(b) the instantaneous velocity at  $t = 1 \text{ sec.}$   
at  $t = 4 \text{ sec.}$   
at  $t = 6 \text{ sec.}$

(c) the acceleration in the intervals  $[0, 2] \text{ s}$ ,  $[3, 5] \text{ s}$